



Dynamics, Energetics, and Parameter Study of Double Support Phase in Biped Gait with a Linear Force Increment

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ABSTRACT

This paper is a study on dynamics of a two-link biped in level walking, focusing on the double support phase (DSP). For a mechanical model with generality in leg mechanism dynamics is undetermined including one degree of redundancy for planar bipeds, two for spatial ones. Unlike the common optimization-based approaches for solving such distribution problem, we consider a more simple assumption that is computationally light. Dynamics and state behavior of the resultant solution is studied on the phase plane as well as in terms of actuation and energy consumption. The effect of variations in the model and motion parameters as well as initial conditions is investigated on determinant features of gait such as the rear leg force, period of DSP, energy consumption, and step length.

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1. INTRODUCTION

Normal gait cycle consists of two rather similar and symmetrical steps. In walking at least one foot is in contact with the ground at every moment, hence, a cycle is divided into two single and two double support phases (SSP&DSP), taking place one after another.

The use of inverted pendulum models to study the bio-mechanics of walking is a common practice. In its simplest form, the inverted pendulum consists of a pointed mass, which models the center of mass of the individual, attached to two straight mass-less segments, the legs. As far as the authors have been able to ascertain the first mention of the term ‘‘inverted pendulum’’ (IP) as a model of the stance phase of walking was by Cavagna et al. [1] although similar concepts can be traced much earlier [2–4]. More recently, the IP has formed the basis of a growing body of work associated with the Dynamic Walking movement [5] which is based on principles first elucidated by Mochon and McMahon [6] and subsequently by Tad McGeer [7,8]. Recent works of this group has tended to focus on the transitions from one step to the next [9–11]. The group, as well as other researchers, have also presented many extended versions of IP models including springs, dampers, telescopic actuators, additional segments and joints [12–16].

The mechanics of the IP mechanism were presented briefly by Anderson and Pandy [17] and gave a brief description of the ground reaction force (GRF) under an IP. A more comprehensive comparison with gait data is done by Buczek et al. [18]. It is also common for investigating upright stability [19-21] or predicting loss of balance during standing or walking [22], for planning or optimization of trajectories for walking [23], to use IP models.

Most of the works focused on the single support phase (SSP) of walking gait and less work has been done in double support phase (DSP). In the next section, we demonstrate that an irreducibly simple, 2D, two-link model, vaguely resembling human legs. This model is the simplest special case of the passive-dynamic models pioneered by McGeer [7]. It has two rigid massless legs hinged at the hip, a point-mass at the hip, and infinitesimal point-masses at the feet. In the following section, we describe the equations governing the dynamics of the mass in DSP. Subsequently, we identify the solutions to the governing equations that correspond to periodic walking. We then report results of numerical simulations. We further explore our model by restricting our attention to the sensitivity analysis to the parameters and examined the effects of variations in the

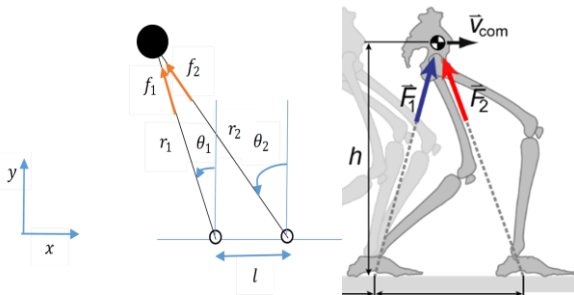


Fig1. Simple models of walking, with forces from each general leg acting on the COM. the distance L between two legs remains constant. The gravity force vector mg is located at the COM, pointing vertically downward. The pressure on the feet can be represented by a double ground reaction force vector F_R & F_F , located at the center of pressure (COM).

parameters on the determinant features of gait like the rear leg force, period of DSP time, the cost of energy and also examined the effect of changing initial condition on the determinant features by drawing the phase plane diagram and examined the range of the phase diagram, that can be divided into four general sections. Finally the net mechanical work done on the center of mass, the positive and negative external work by the rear and front legs will be calculated and discussed.

2. METHODS

A. Model

Walking study like any other problems, need a good model before. The assumed model should be close to the human model in terms of structure, kinematics and dynamical structure, as far as possible and in addition to the ability to make changes according to the type of problem and the appropriate response to those changes.

We consider simple model of the leg forces acting on the COM during walking. we model the COM as a pointed mass located at the hip, assuming it to be the primary load during walking. We therefore neglect the mass of the legs and rotational inertia of them. the length of the foot is variable at any moment, which is represented by a sliding joint. Fig1.

We initially consider an ideal inverted pendulum gait, concentrating on the double-support phases, with two legs contacting the ground, respectively. We assume that there is no actuator at the hip and in the contact area of the legs with the ground (in these points we have a simple joint without torque) and also, assume that there isn't any slip in the contact area of the legs with the ground.

This model can be studied analytically, yet it captures major characteristics of walking. it is also easily augmented with more realistic anthropomorphic features. legs behaving like a rigid inverted pendulum during double-support phases.

One consequence of the above assumptions is that the structure of the foot, the number of links and joints and the type of actuator (s) is not important. The problem can completely be described by the movement of the whole-body COM.

The mechanical work performed on the center of mass is equal to cumulative time-integral of the dot product of two vectors: the ground reaction force vector located at the center of pressure (COM) and the hip distance to the point of contact of the leg. the amount of work done or the consumed energy used to move in the period of walking is considered as the cost function and meaning of the energy consumption is when the force and the length of the legs are both in one direction, otherwise the energy is only wasted, so that in the entire cycle, the amount of work done is equal to zero, positive external mechanical work is calculated by restricting the integral to the intervals over which the integrand is positive (denoted by the domain POS). we found the positive external work performed by the rear leg W_{Rear}^+ and front leg W_{Front}^+ using F_R and F_F ; respectively.

$$W_{Rear}^+ = \int_{POS} F_R \cdot dr_R \quad (1)$$

$$W_{Front}^+ = \int_{POS} F_F \cdot dr_F \quad (2)$$

Correspondingly, the negative external mechanical work is equal to the cumulative time-integral of external mechanical power, restricting the integral to the intervals over which the integrand is negative (denoted by the domain NEG). We found the negative external work performed by the rear leg W_{Rear}^- and front leg W_{Front}^- using F_R and F_F , respectively. We calculated the positive and negative external mechanical work performed by each limb over particular time intervals (double support phase) by restricting the integration of the positive and negative external mechanical work to these time intervals. the total positive W_{tot}^+ or negative W_{tot}^- external mechanical work is the sum of the positive or negative external mechanical work performed by each limb:

$$W_{tot}^+ = W_{Rear}^+ + W_{Front}^+ \quad (3)$$

$$W_{tot}^- = W_{Rear}^- + W_{Front}^- \quad (4)$$

B. Kinematics

Polar coordinate system is used for to describe the motion. We also convert the r derivation respect to time to derivation of r to $\dot{\theta}$:

$$\dot{r} = r' \dot{\theta} \quad (7)$$

$$\ddot{r} = r'' \dot{\theta}^2 + r' \ddot{\theta} \quad (8)$$

Where $r' = \frac{\partial r}{\partial \theta}$. we expressed the velocity and acceleration of the mass in the polar coordinate system:

$$\vec{v} = \dot{\theta} \left[(r') \hat{r} + (r) \hat{\theta} \right] \dot{\theta} \begin{bmatrix} r' \\ r \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{\theta} \end{bmatrix} \quad (9)$$

$$\vec{a} = (r' \ddot{\theta} + (r'' - r) \dot{\theta}^2) \hat{r} + (r \ddot{\theta} + 2r' \dot{\theta}^2) \hat{\theta} \quad (10)$$

$$= \begin{bmatrix} \ddot{\theta} & \dot{\theta}^2 \end{bmatrix} \begin{bmatrix} r' & r \\ r'' - r & 2r' \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{\theta} \end{bmatrix}$$

And also dimension of the velocity is determined by:

$$V := |\vec{v}| = |\dot{\theta}| \sqrt{r'^2 + r^2} \quad (11)$$

C. Kinetics

DSP is modeled as a closed-link since both legs are in contact with the ground. The model has joints at hip, ankle, and toe during the initial DSP. The equations of motion are generated through the Newton's principles. Newton's equation of motion for the whole body in the DSP phase is given by:

$$\ddot{r}_R - r_R \dot{\theta}_R^2 + g \cos(\theta_R) = \frac{F_R}{m} + \frac{F_F}{m} \cos(\theta_F - \theta_R) \quad (12)$$

$$2\dot{r}_R \dot{\theta}_R + r_R \ddot{\theta}_R - g \sin(\theta_R) = \frac{F_F}{m} \sin(\theta_F - \theta_R) \quad (13)$$

We assume the r_R and θ_R alignment as a reference and established r_F and θ_F by the relationship with them (23-25). where m is body mass that is centered on the hip position, r_R and r_F are respectively, hip distance to the point of contact of the Rear leg and front leg with the ground. θ_R is the angle between r_R and vertical alignment (CCW) and θ_F is the angle between r_F and vertical alignment (CCW). Finally, F_R is the ground reaction force from the rear (trailing) leg and F_F is the ground reaction forces from the front leg. In this equation, the number of unknown variables is more than the number of equations. Thus, an inverse dynamics model for DSP phase of

gait is developed to study segmental contributions to the ground reaction force (F_R & F_F) which are usually used as inputs to compute θ_R .

D. Solution algorithm

In order to solve equations (12-13), we assume that the hip moves in the constant height y_0 :

$$r_R(\theta_R) = \frac{y_0}{\cos(\theta_R)} \quad (14)$$

The dynamics model can be written as:

$$\frac{y_0}{\cos(\theta_R)} \ddot{\theta}_R + 2y_0 \frac{\sin(\theta_R)}{\cos^2(\theta_R)} \dot{\theta}_R^2 - g \sin(\theta_R) = \frac{F_F}{m} \sin(\theta_F - \theta_R) \quad (15)$$

$$y_0 \frac{\sin(\theta_R)}{\cos^2(\theta_R)} \ddot{\theta}_R + \left(2 \frac{y_0}{\cos(\theta_R)} \tan^2(\theta_R) \right) \dot{\theta}_R^2 + g \cos(\theta_R) = \frac{F_R}{m} + \frac{F_F}{m} \cos(\theta_F - \theta_R) \quad (16)$$

In these two equations, we have three unknowns F_R , F_F and θ_R as a function of the independent variable t .

These equations have an uncertain answer, which the distribution of force between two legs is not unique. In order to make the solution unique, optimization (e.g. energy) may be used. However, optimization techniques are basically expensive in terms of computation cost, and hence are hard to realize in real-time problems. Here we take an assumption which is close to what happens in the reality and discuss the results

Assume continuity of force on time during SSP and DSP, we consider that the force of the front foot ($F_F(t = t_f)$) at the end of DSP was, be equal to the force at the start of the SSP (F_{iSSP}). Also by regarding that the front foot was separated from the ground before the start of DSP (during all of the SSP time) we consider the front foot force equal to zero at the start of DSP. We set F_F between these two values linearly respect to time:

$$F_F(t) = \left(\frac{t}{t_f} \right) F_{iSSP} \quad (17)$$

Where f_{iSSP} is the initial force of stance leg at SSP, whose value is obtained from the dynamic solution in a SSP and using Equation (15), we arrive to a differential equation with the two unknowns, θ_R and t_f :

$$\frac{y_0}{\cos(\theta_R)} \ddot{\theta}_R + 2y_0 \frac{\sin(\theta_R)}{\cos^2(\theta_R)} \dot{\theta}_R^2 - g \sin(\theta_R) - \frac{\left(\frac{t}{t_f} \right) F_{iSSP}}{m} \sin(\theta_F - \theta_R) = 0 \quad (18)$$

The answer of the equation must be satisfying the final condition: $\theta_R(t = t_f) = \theta_{Rf}$. now we have two equations with two unknowns θ_R and t_f . In order to solve equation (17) we guess t_f and stablish $\theta_R(t)$. We consider an error function named $\varepsilon(t)$ as difrence between $\theta_R(t)$ at t_f and θ_{Rf} at the final condition (θ_{Rf}):

$$\varepsilon(t) = \theta_R(t = t_f) - \theta_{Rf} \quad (19)$$

We plot the diagram $\varepsilon(t)$ against to t_f and determine the time where $\varepsilon(t)$ is zero. The obtained time is the period of the DSP, at least, we can calculate the force of the Rear leg (F_R) by the equation below:

$$F_R = m * y_0 * \frac{\sin(\theta_R)}{\cos^2(\theta_R)} \ddot{\theta}_R + \left(2 * m * \frac{y_0}{\cos(\theta_R)} \tan^2(\theta_R) \right) \dot{\theta}_R^2 + m * g \cos(\theta_R) - \left(\frac{t}{t_f} \right) f_{iSSP} * \cos(\theta_F - \theta_R) \quad (20)$$

We determined the center of mass average velocity using the vector of the center of mass displacement and t_f (the period of the DSP):

$$s = r_R(\theta_{Rf}) \sin(\theta_{Rf}) - r_R(\theta_{Ri}) \sin(\theta_{Ri}) \quad (21)$$

$$V_{avg} = \frac{s}{t_f} \quad (22)$$

E. Constraints in double support phase(DSP)

E.1. Kinematics constraints in double support phase(DSP):

$$-r_R \sin(\theta_R) + r_F \sin(\theta_F) = L \quad (23)$$

$$r_R \cos(\theta_R) = y \quad (24)$$

$$r_F \cos(\theta_F) = y \quad (25)$$

in this model, the distance L between two legs remains constant. we assume that (y) is the position of hip height.

E.2. Periodic constraints in double support phase(DSP)

We assume that the walking cycle is a periodic, that is, a period of time ($2T$), except the horizontal displacement, the rest of the parameters at each moment and the next period of time is equal. We assume that the walking cycle is symmetric. It means, every story that one leg has over a complete cycle, the other leg will have the same half-cycle difference time, except for the horizontal component of the places that has a pure displacement. we assume the height of hip at the start of DSP be equal by it at the end.

$$r_R(\theta_{Rf}) \cos\theta_{Rf} = r_R(\theta_{Ri}) \cos\theta_{Ri} \quad (26)$$

We also, consider the velocity vector of hip at the start of DSP be equal by it at the end, (it contains direction and magnitude).

$$\vec{V}(\theta_f) = \vec{V}(\theta_i) \quad (27)$$

$$\frac{V_y(\theta_f)}{V_x(\theta_f)} = \frac{V_y(\theta_i)}{V_x(\theta_i)} = \frac{r_1'(\theta_{if}) \cos\theta_{if} - r_1(\theta_{if}) \sin\theta_{if}}{-r_1'(\theta_{if}) \sin\theta_{if} - r_1(\theta_{if}) \cos\theta_{if}} = \frac{r_1'(\theta_{ii}) \cos\theta_{ii} - r_1(\theta_{ii}) \sin\theta_{ii}}{-r_1'(\theta_{ii}) \sin\theta_{ii} - r_1(\theta_{ii}) \cos\theta_{ii}} \quad (28)$$

$$V(\theta_f) = V(\theta_i) = \theta_i(\theta_f) \sqrt{r_1'(\theta_f)^2 + r_1(\theta_f)^2} = \theta_i(\theta_i) \sqrt{r_1'(\theta_i)^2 + r_1(\theta_i)^2} \quad (29)$$

E.3. Kinetics constraints in double support phase(DSP)

Feasible motions of the biped model are defined by specifies interaction conditions between the stances foot and the

ground. the vertical component of ground reaction forces must remain positive during the motion due to unilaterality of contact. this condition means that the foot is not stuck on the ground and the ground can only push it. therefore, the unilaterality condition is expressed by equation (20).

$$\forall t \in [t_i t_f] \quad 0 < F_n^{min} \leq F_n(t) \quad (30)$$

The latter condition can be expressed by ignoring the slide of the foot on the ground in equation (21).

$$\forall t \in [t_i t_f] \quad |F_t(t)| \leq \mu F_n(t) \quad (31)$$

3. RESULTS

A computer program in MATLAB (Mathworks, Natick, MA, 2014) environment solves DSP dynamics for the model parameters given in Table 1 and initial and final conditions given in Table 2.

TABLE 1. Parameters values used to solve the dynamics

parameter	value
Gravity gradient(g)	10 m/s ²
Height	1 m
Mass(m)	70 kg
Distance between two legs(L)	0.8 m

TABLE 2. Initial and final condition of DSP:

initial condition of DSP		final condition of DSP	
θ_R	$= \theta_{F_{SSP}} = -\frac{\pi}{10}$ $= -0.3142rad$	θ_F	$= \theta_{i_{SSP}} = \frac{\pi}{10} = 0.3142rad$
$\dot{\theta}_R$	$\dot{\theta}_{F_{SSP}} = -1.0854$	$\dot{\theta}_F$	$\dot{\theta}_{i_{SSP}} = -1.0854$
θ_F	$\tan^{-1}(\tan(\theta_R) + \frac{L}{y_0})$ $= 0.4435rad$	θ_R	$\tan^{-1}(\tan(\theta_F) - \frac{L}{y_0})$ $= -0.4435rad$

Due to the topics discussed at prat (D) we plot the diagram $\varepsilon(t)$ against to t_f and determine the time where $\varepsilon(t)$ is zero. the obtained time is the period of the DSP. we established at $t_f = 0.1182$ (s), $\theta_R(t = t_f) = \theta_{Rf}$ and the error be almost zero.

Fig.2 shows Respectively, the angle, the velocity and the acceleration of rear leg ($\theta_R, \dot{\theta}_R, \ddot{\theta}_R$) against the time.as expected $\theta_R(t = 0) = -0.3142 = -\frac{\pi}{10} rad$ that is initial condition of DSP and must be equal to the angle of front leg at the end of SSP ($\theta_{F_{SSP}} = -\frac{\pi}{10} rad$) . the rear leg angle at the end of DSP should be equal to the amount for which the angle of front leg at the end of DSP is equal to angle of stance(front) leg at the start of SSP ($\theta_F(t = t_f) = \theta_{i_{SSP}} = \frac{\pi}{10} = 0.3142rad = 18deg$) we established $\theta_R(t = t_f = 0.1182$ (s)) $= -0.4435 rad$ that satisfy this condition.(Fig2-3).

Fig.4. illustrate: F_R the ground reaction forces from the rear(trailing) leg and F_F is the ground reaction forces from the front leg. by regarding that the front foot was separated from the ground before the start of DSP (during all of the time at SSP). it is equal to zero at ($t = 0$) and the force of the front foot ($F_F(t = t_f)$) at the end of DSP is equal to the force at the start of the SSP ($f_{i_{SSP}} = 720.4126$). we set F_F between these two values linearly respect to time (equation (17)), Where $f_{i_{SSP}}$ is the initial force of stance leg at SSP, whose value is obtained from the dynamic solution in SSP.

Due to the topics discussed at prat (F) we established the force at the rear leg compressive and $F_R \geq 0$, and such conditions satisfy during the entire DSP (assumption 28), then the assumption is valid. it is equal to $F_R(t = 0) = 684.0481$ and $F_R(t = t_f) = 16.4505$. as seen the relationship between F_R and t is almost linear and the fitted equation on the graph is (Adjusted R-square= 0.9993):

$$F_R(t) = -6087 * t + 747.2 \quad (32)$$

Also, we obtain the velocity at the end of DSP equal to the velocity at the start of it. $V_i = -1.2, V_f = -1.2174, V_{avg} = -1.21 = m/s^2$. we obtained front leg performs negative work to redirect the center of mass velocity ($W_{FF} = -19.0013$) for $L = 0.8$, while simultaneously, the rear leg performs positive work to replace the lost energy ($W_{FR} = 20.4749$).

4. DISCUSSIONS

The aim of this study was to further investigate what insights simple IP based models at DSP can give us into the mechanisms that drive human walking. we discussed about parameters sensitivity at the results. first, we examine the influence of changing the length of distance between two legs (L) on the determinant features of gait, focusing on the rear leg force and period of DSP time and etc.

We consider the rear leg angle at the beginning of DSP be equal to front leg angle at final condition in SSP and also the front leg angle at final condition in DSP must be equal front leg angle at initial condition in SSP. (Fig5). as expected, by increasing the distance between two legs (L) the time required for rear angle to reach to final condition increased and the rear angle during this time are in decline, means that absolute value of it increased. also, the angular velocity and acceleration of rear leg increased. and the same results are true for the front leg(Fig6). As discussed in the previous section, an inverse dynamics model for the double support (DSP) phase of gait is developed to study segmental contributions to the ground reaction force (F_R & F_F).

so far, the COM moves in a level path at constant velocity. the relationship between angle of legs and r (distance between hip and contact point of the leg with the ground), is independent of L , while r increases against time by increase distance L , but as seen in Fig. 8, at $L = 1-1.2$ m by increasing the time the length of (r) begins to decrease. this range is the same range that follows, which is also unusual in terms of work (cost of energy). By regarding to these results, we can discuss about several cases, one of them is an upper and lower limit must be acceptable for the distance between two legs (L), that in this problem allowable L is in rang of (0.7-0.9) and second case, that must be considered, is relationship between speed and step length in walking. the other case is about initial condition like velocity of hip. by given different

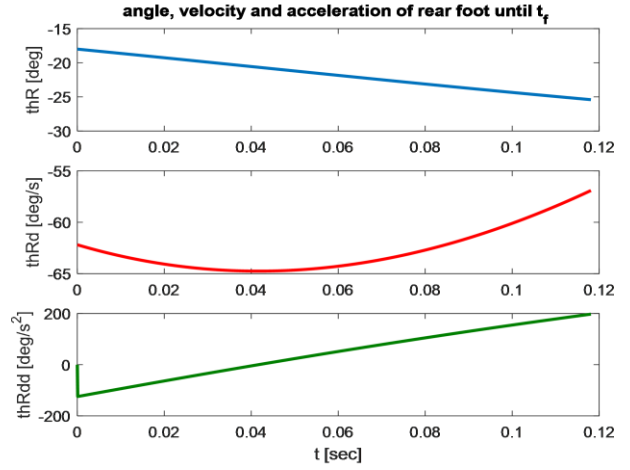


Fig.2. the angle, velocity and acceleration of rear leg ($\theta_R, \dot{\theta}_R, \ddot{\theta}_R$) against the time ($t = t_f$). $\theta_R(t = 0) = -18$ deg that is initial condition of DSP and must be equal to the angle of front leg at the end of SSP ($\theta_{F_{SSP}} = -18$ deg). the rear leg angle at the end of DSP should be equal to the amount for which the angle of front leg at the end of DSP is equal to angle of stance(front) leg at the start of SSP ($\theta_F(t = t_f) = \theta_{i_{SSP}} = 18$ deg) we established $\theta_R(t = t_f = 0.1182$ (s)) = -25.41 deg, that satisfy this condition.

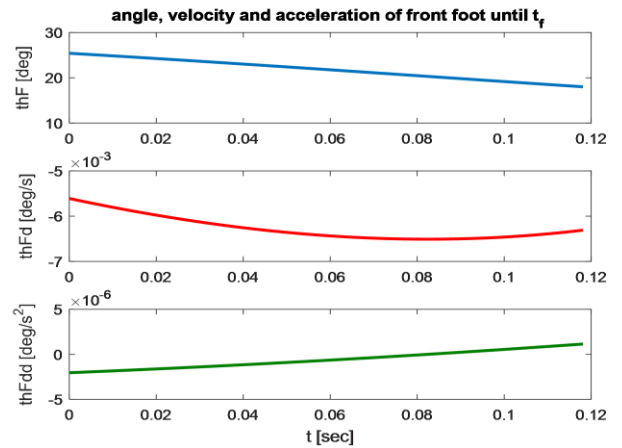


Fig.3. the angle, velocity and acceleration of front leg ($\theta_F, \dot{\theta}_F, \ddot{\theta}_F$) against the time ($t = t_f$). $\theta_F(t = t_f) = 18$ deg that we consider it as a final condition of DSP.

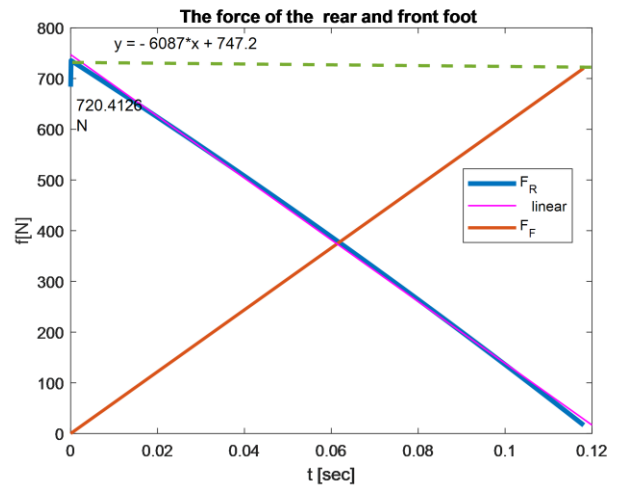


Fig.4. F_R the ground reaction forces from the rear leg and F_F is the ground reaction forces from the front leg. the front foot (F_F) was separated from the ground before the start of DSP. it is equal to zero at ($t = 0$) and the force of the front foot ($F_F(t = t_f)$) at the end of DSP is equal to the force at the start of the SSP ($f_{i_{SSP}} = 720.4126$). we set F_F between these two values linearly respect to time and established F_R by using equation (20). as seen in Fig this can be considered with a very good approximation as a linear equation.

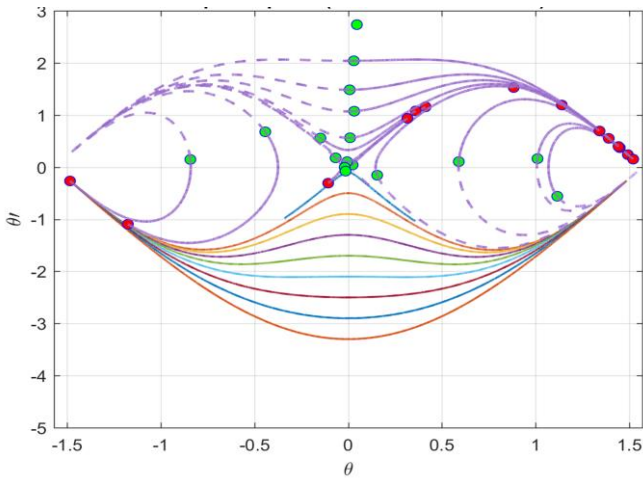


Fig.7.in the main phase there are a series of trajectory for a particular initial condition, which makes it possible to reach to the final condition. with another initial condition may not be possible to reach the final condition and the path does not lead to the final condition. according to fig7, the range of motion of the model is in the first and fourth quarters, the variations of angle are in the range of $-\pi / 2$ to $+\pi / 2$. according to the points mentioned, the range of the phase diagram can be divided into four general sections.

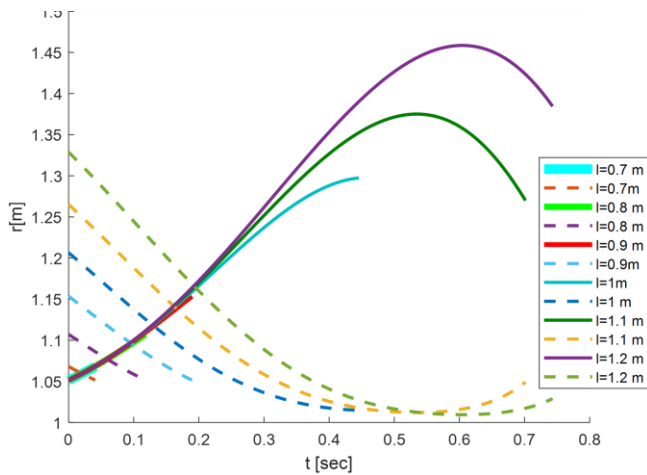


Fig.8. The effect of variant L on r (distance between hip and contact point of the leg). r in the rear leg increases against time by increase distance L, but as seen in Fig. 8, at $L=1-1.2$ m by increasing the time the length of (r) begins to decrease and r in the front leg decreases against time by increase distance L, but as seen in Fig. 8, at $L=1-1.2$ m by decreasing the time the length of (r) begins to increase. this range is the same range that follows, which is also unusual in terms of work (cost of energy)

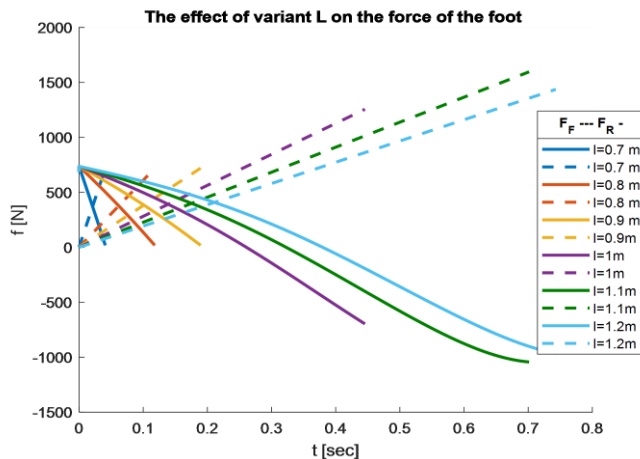


Fig.9.The effect of variant L on the force of rear& front leg.

named phase-plane diagram (Fig7), we find that there are the series of trajectory for a particular initial condition, which makes it possible to reach to the final condition. with another initial condition may not be possible to reach the final condition and the path does not lead to the final condition. According to Fig7, the range of motion of the model is in the first and fourth quarters, the variations of angle are in the range of $-\pi / 2$ to $+\pi / 2$. also, due to the same direction (the positive direction of x), therefore, the angular velocity must also be negative. Now, according to the points mentioned, the range of the phase diagram can be divided into four general sections: **The 1st range:** if the initial condition of the problem placed in the first range, motion is in the first quarter. that means the center of mass moves towards the fourth quarter, but in the same quarter the velocity gets zero and returns to its stable equilibrium point $(\pi / 2.0)$. **The 2nd range:** if the initial condition of the problem placed in the second range, the motion will be the same as the previous one, with the difference that the motion here is in the fourth quarter and the center of mass goes to the stable equilibrium point $(-\pi / 2.0)$. **The 3rd range:** if the initial conditions are placed in the third, at three points from the path the acceleration changes to zero. In this case, the minimum velocity occurs at two angles. At the end, the center of mass moves toward the equilibrium point $(-\pi / 2.0)$. **The 4th range:** the motion in the fourth range is similar to the motion in the third range, with the difference that, when the angle is zero the acceleration is changed the sign.

During double support both legs produce force (denoted by $\sim FR$ and $\sim FF$) directed at the COM from the center of pressure on the ground. this force both supports body weight and performs positive and negative work to maintain the level COM path. The flattened trajectory requires a great deal of work during double support. by increase in the distance between two legs, more forces are needed in both legs as expected.

For walking on level ground, however, it is unclear why mechanical work is required. There is no dissipative load external to the body as for rowing or cycling, nor is net work performed against gravity as for slope walking. Perhaps the body and limbs themselves act as a mechanical load. There are a variety of methods to quantify the mechanical work performed on the body and limbs [1], but these neither predict nor explain where and why mechanical energy is dissipated. Inverted pendulum model of walking predict that work is not needed within each step, but rather between steps .in bipeds, double support can be modeled as an inverted pendulum that, negative work is performed in the collision front leg to the ground, that redirects the center of mass velocity from and positive work is required to restore the energy lost. We obtained front leg performs negative work to redirect the center of mass velocity ($w_{FF}=-19.0013$) for $L=0.8$, while simultaneously, the rear leg performs positive work to replace the lost energy ($w_{FR}=20.4749$).by increase L the value of FR and FF also increased, but in range $L=1.2m$ and more, the values of the work are greatly increased and it is not optimal.

initial conditions, there are certain trajectory of motion, that

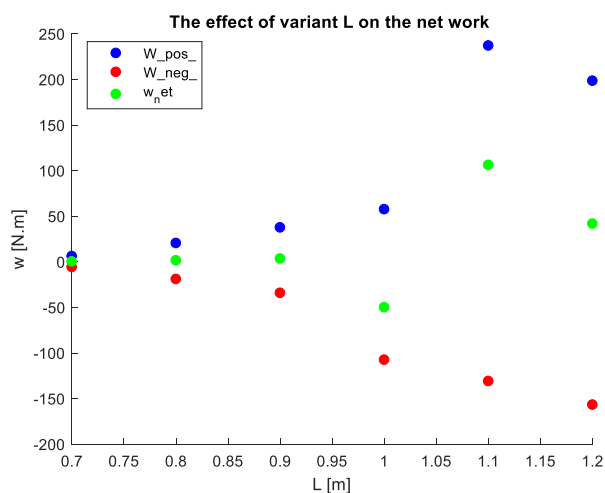


Fig.10.The effect of variant L on the positive and negative and net work

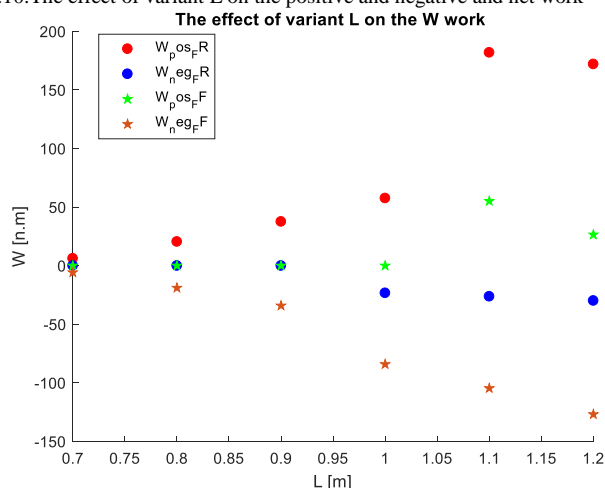


Fig.11. The effect of variant L on the positive and negative work of the leg

We expect that net work was equal to zero at the period of time in DSP but as seen because of waste of energy it is not zero. also by increasing the distance between two legs it also greatly increases and it is not cost effective and we established optimal L is at the range of $L=0.7m$.

5. CONCLUSION

The aim of this study was to further investigate what insights simple IP based models at DSP can give us into the mechanisms that drive human walking. we describe the equations governing the dynamics of the mass while two feet is on the ground that called double support phase. Subsequently, we identify the solutions to the governing equations that correspond to periodic walking. We then report results of numerical simulations. we discussed about parameters sensitivity at the results. First, we examine the influence of changing the length of distance between two legs(L) on the determinant features of gait, focusing on the rear leg force and period of DSP time and etc. In summary, we examined during the double support phase, a substantial amount of positive and negative mechanical work is performed simultaneously by the trailing and leading legs, respectively. Negative work by the leading leg is used to change the direction of the center of mass velocity between DSP Positive work is necessary to restore lost energy.

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