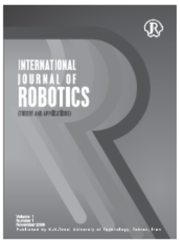

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Hybrid Control to Approach Chaos Synchronization of Uncertain DUFFING Oscillator Systems with External Disturbance

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ABSTRACT

This paper proposes a hybrid control scheme for the synchronization of two chaotic Duffing oscillator system, subject to uncertainties and external disturbances. The novelty of this scheme is that the Linear Quadratic Regulation (LQR) control, Sliding Mode (SM) control and Gaussian Radial basis Function Neural Network (GRBFNN) control are combined to chaos synchronization with respect to external disturbances. By Lyapunov stability theory, SM control is presented to ensure the stability of the controlled system. GRBFNN control is trained during the control process. The learning algorithm of the GRBFNN is based on the minimization of a cost function which considers the sliding surface and control effort. Simulation results demonstrate the ability of the hybrid control scheme to synchronize the chaotic Duffing oscillator systems through the application of a single control signal.

1-Introduction

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades. Chaotic phenomena can be found in many scientific and engineering fields such as biological systems, electronic circuits, power converters, chemical systems, and so on [1].
Since the synchronization of chaotic dynamical systems has been observed by Pecora and Carroll [2] in 1990, chaos synchronization has become a topic of great interest [3-5]. Synchronization phenomena have been reported in the recent literature. Until now, different types of synchronization have been found in interacting chaotic systems, such as complete synchronization, generalized synchronization, phase synchronization and anti-phase synchronization [6-8], etc.
In this study, a hybrid control scheme is applied to chaos synchronization. Two identical chaotic system such as Duffing oscillator have been considered as the master and the slave systems. The slave system has been subjected to model uncertainty and external disturbances. To achieve the presented goal, some control techniques such as LQR, SMC , GRBFNN and hybrid control have been designed.
This paper is organized as follows. In section II, the dynamics of a nonlinear duffing system is explained. The synchronization problem for nonlinear duffing systems is described in section III. In section IV, the material and methods are explained. In this section, LQR control is designed. Also, SM control is designed. $GRBF_{NN}$ control and the learning algorithms of this controller are presented. Moreover, hybrid control for synchronizing of nonlinear duffing systems is presented. Finally, to show the effectiveness of these control methods for synchronization, simulations are presented in section V. At the end, the paper is concluded in section VI.

2-Duffing Oscillator System

Consider a second-order chaotic system such as well known Duffing's equation describing a special nonlinear circuit or a pendulum moving in a viscous medium under control [9]:

$$\ddot{x} = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(\omega\, t) \qquad (1)$$

where p, p_1, p_2 and q are real constants . t is the time variable and ω is the frequency.

Given the states $x_1 = x$ and $x_2 = \dot{x}$, then the Eq. (1) can be rewritten as follow:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1 x_1 - p_2 x_1^3 + q\cos(\omega\, t) \end{cases} \qquad (2)$$

This system exhibits complex dynamics and has been studied by [9]. The constant values of Eq. (2) are $p = 0.4$, $p_1 = -1.1$, $p_2 = 1$, $q = 0.62$ and $\omega = 1.8$.

Fig. 1 and Fig. 2 illustrate the irregular motion exhibited by this system and initial conditions of $(x_1, x_2) = (1, -1)$.

In the next section, the problem of synchronizing two identical Duffing system with different initial conditions is described.

Notice that model uncertainty and external disturbances appear in the slave system.

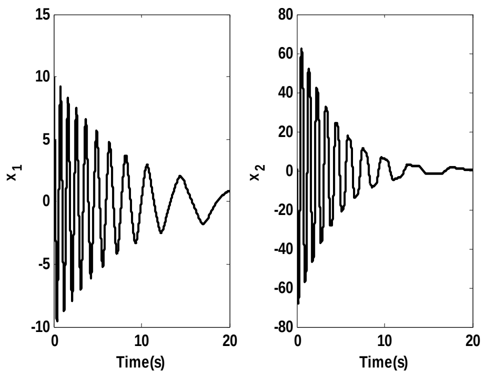

Fig.1: Time series of x_1 and x_2 .

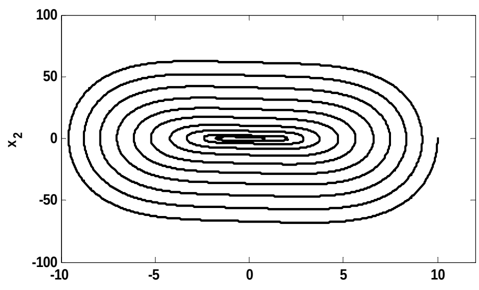

Fig. 2: Phase plane trajectory of a chaotic Duffing oscillator system.

3-Synchronization Problem

Consider two coupled, chaotic gyro systems are as following:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1 x_1 - p_2 x_1^3 + q\cos(\omega\, t) = g(x_1, x_2, \omega, t) \end{cases} \qquad (3)$$

And

$$\begin{cases} y_1 = y_2 \\ y_2 = py_2 - p_1 y_1 - p_1 y_1^3 + q\cos(\omega t) + \Delta f(y_1, y_2) \\ + d(t) + u(t) = g(y_1, y_2, \omega, t) + \Delta f(y_1, y_2) + d(t) + u(t) \end{cases} \qquad (4)$$

where $u \in R$ is the control input, $\Delta f(y_1, y_2)$ is an uncertainty term representing the un-modeled dynamics or structural variation of the system is given in Eq. (4) and $d(t)$ is the time-varying disturbance.

In general, the uncertainty and the disturbance are assumed to be bounded as follows:

$\left|\Delta f(y_1, y_2)\right| \le \alpha$ And $\left|d(t)\right| \le \beta$.

where α and β are positive constant values.

The systems described in Eq. (3) and Eq. (4) correspond to the master system and the slave system, respectively, and the objective of the current control problem is to design an appropriate control signal $u(t)$ such that for any initial conditions of the two systems, the behavior of the slave converges to that of the master. Defining the state errors between the master and slave systems as:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \end{cases} \qquad (5)$$

Then, the dynamics equations of these errors can be determined by subtracting Eq. (3) from Eq. (4) as follow:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = g(y_1, y_2, \omega, t) - g(x_1, x_2, \omega, t) + \Delta f(y_1, y_2) + d(t) + u(t) \end{cases} \qquad (6)$$

4- Material and Methods

4-1-Linear Quadratic Regulation Control

This method determines the state feedback gain matrix that minimizes J in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that together guarantee a stable system. After linearization of the system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad (7)$$

Determine the control effort $u(t)$ as:

$$u(t) = -kx(t) \qquad (8)$$

And $k = R^{-1}B^T X$, where X is obtained from Matrix Differential Riccati Equation (MDRE), MDRE is as follows:

$$-\dot{X}(t) = A^T(t)X(t) + X(t)A^T(t) + Q(t) - X(t)B(t)R^{-1}B^T(t)X(t) \qquad (9)$$

So in order to minimize the performance index,

$$J = \frac{1}{2}\int_0^\infty (e^T Q e + u^T R u)dt \qquad (10)$$

where Q and R are the positive definite Hermitian or real symmetric matrices.

Note that the second term on the right hand side accounts for the expenditure of the energy on the control efforts, the matrix of Q and R determine the relative importance of the error and the expenditure of this energy [10].

4-2-Sliding surface and sliding mode control

Using an SM control method to synchronize the chaotic Duffing oscillator system, involves two basic steps;

(1) Selecting an appropriate sliding surface such that the sliding motion on the sliding manifold is stable.

(2) Establishing a robust control law which guarantees the existence of the sliding manifold $S(t)=0$ even in the event of uncertainties.

The sliding surface is defined as [11]:

$$S(t)=e_2(t)+\delta\, e_1(t) \tag{11}$$

where δ is a real positive constant. The rate of convergence of the sliding surface is governed by the value assigned to parameter δ. The first derivative of (10) with respect to time is:

$$\dot{S}(t)=\dot{e}_2(t)+\delta\, \dot{e}_1(t) \tag{12}$$

Substituting the Eq. (6) into Eq. (12):

$$\dot{S}(t)=g(y_1,y_2,\omega,t)-g(x_1,x_2,\omega,t)+\Delta f(y_1,y_2)+d(t)+u(t)+\delta\, e_2(t) \tag{13}$$

Define a Lyapunov function as:

$$V=\frac{1}{2}S^2 \tag{14}$$

Differentiating Eq. (14) with respect to time we have:

$$\dot{V}=S\dot{S} \tag{15}$$

Substituting Eq. (13) into (15):

$$\dot{V}=S\left[g(y_1,y_2,\omega,t)-g(x_1,x_2,\omega,t)+\Delta f(y_1,y_2)+d(t)+u(t)+\delta\, e_2(t)\right] \tag{16}$$

Let

$$u(t)=-\eta\,\mathrm{sgn}(S)+g(x_1,x_2,\omega,t)-g(y_1,y_2,\omega,t)-\delta\, e_2(t) \tag{17}$$

where η is a positive constant and $\eta>\alpha+\beta$. Then

$$\dot{V}=-\eta|S| \tag{18}$$

Since $\eta>\alpha+\beta$, the reaching condition ($S\dot{S}<0$) is always satisfied. Thus, the proof is achieved. An appropriate value of η is chosen not only to quicken the time of reaching the sliding mode motion which has a good robustness to the system uncertainties, but also to reduce the system chattering.Therefore, this implies that the sliding surface be chattering in a finite time and the SM controller is used for synchronizing the chaotic Duffing oscillator systems. Thus the error state trajectories converge to the sliding surface $S(t)=0$.

4-3-GRBF Neural Network

The $GRBF_{NN}$ can be considered as one layer feed forward neural network with nonlinear element. The $GRBF_{NN}$ output can perform the mapping according to:

$$f(z)=\sum_{j=1}^{n} w_j G_j(z_j,m_j,\sigma_j) \tag{19}$$

where $z=[z_1,z_2,...,z_n]^T\in R^n$ is the input vector, $G_j(z_j,m_j,\sigma_j)\in R^n$, $j=1,2,...,n$ are the Gaussian radial basis functions, m_j is the mean value of the Gaussian function, $\sigma_j\in R$ is the spread of Gaussian function n is the number of neurons.

Each Gaussian radial basis function can be represented by:

$$G_j(z_j,m_j,\sigma_j)=\exp\left(\frac{z_j-m_j}{\sqrt{2}\sigma_j}\right)^2 \tag{20}$$

$GRBF_{NN}$ can be used for synchronizing the chaotic Duffing oscillator systems. To achieve this goal, it is assumed that the output of $GRBF_{NN}$ is the control effort of the synchronization, then $u(t) = f$.

4-4-Learning Algorithm of $GRBF_{NN}$

The goal is to minimize the following cost function:

$$E(k)=S(k)\dot{S}(k)+\frac{1}{2}(u^T(k)u(k)) \tag{21}$$

where $S(k)$ is the sliding surface that was described in the previous section. By using the BP algorithm, the weighting vector of the $GRBF_{NN}$ is adjusted such that the cost function defined in Eq. (21) is less than designed. The well-known algorithm may be written briefly as:

$$w(k+1)=w(k)+\gamma\left(-\frac{\partial E(k)}{\partial w}\right) \tag{22}$$

where γ and w represent the learning rate and tuning parameter of RBFNN. The gradient of $E(.)$ in Eq. (22) with respect to a weighting w is:

$$\frac{\partial E(k)}{w}=S(k)\frac{\partial \dot{S}(k)}{\partial f}\frac{\partial f(k)}{\partial w}+u(k)\frac{\partial f(k)}{\partial w} \tag{23}$$

where

$\frac{\partial f(k)}{\partial w}=G(z,m,\sigma)$ and $\frac{\partial \dot{S}(k)}{\partial f}=1$.Then,

$$\frac{\partial E(k)}{w}=(S(k)+u(k))G(z,m,\sigma) \tag{24}$$

Substituting the Eq. (24) into the (22):

$$w(k+1)=w(k)+\gamma(S(k)+u(k))G(z,m,\sigma) \tag{25}$$

4-5-Hybrid Control

The structure of hybrid control to synchronize the chaotic Duffing oscillator system is shown in Fig. 3.The total control effort is computed as follows:

$$u(t) = u_{LQR}(t) + m(t)u_{SM}(t) + (1 - m(t))u_{GRBF_{NN}}(t) \quad (26)$$

where $u_{LQR}(t)$ is the LQR control, $u_{SM}(t)$ is the SM control and $u_{GRBF_{NN}}(t)$ is the $GRBF_{NN}$ control. The function $m(t)$ allows a smooth transition between the SM controller and the $GRBF_{NN}$ controller, based on the location of the system state:

$$\begin{cases} m(t) = 0 & e(t) \in A_d \\ 0 < m(t) < 1 & otherwise \\ m(t) = 1 & e(t) \in A_c \end{cases} \quad (27)$$

where the regions might be defined as in Fig. 4.

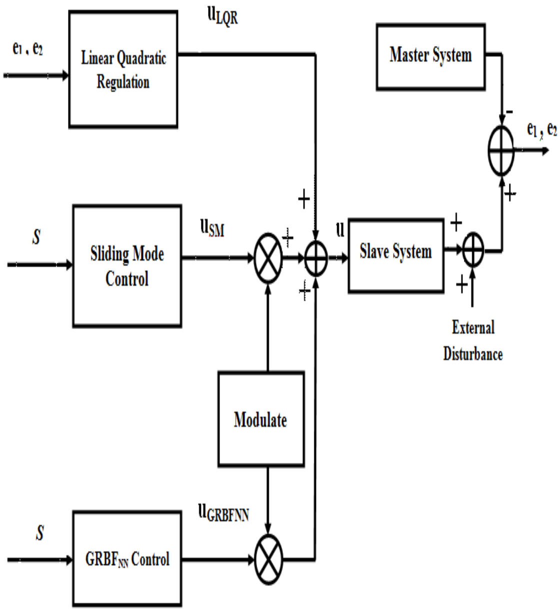

Fig. 3: oscilator Structure of hybrid control to synchronize of chaotic Duffing system.

The SM controller is used to keep the error states in a region where the neural network can be accurately trained to achieve optimal control. The SM controller is turned on (and the neural controller is turned off) whenever the system error states drifts outside this region. The combination of controllers produces a stable system, which adapts to optimize performance.

5-Simulation results

The parameters of chaotic Duffing oscillator systems are specified as follows:
$p = 0.4$, $p_1 = -1.1$, $p_2 = 1$, $q = 0.62$ and $\omega = 1.8$, which, as shown in section 2, give rise to a chaotic state.

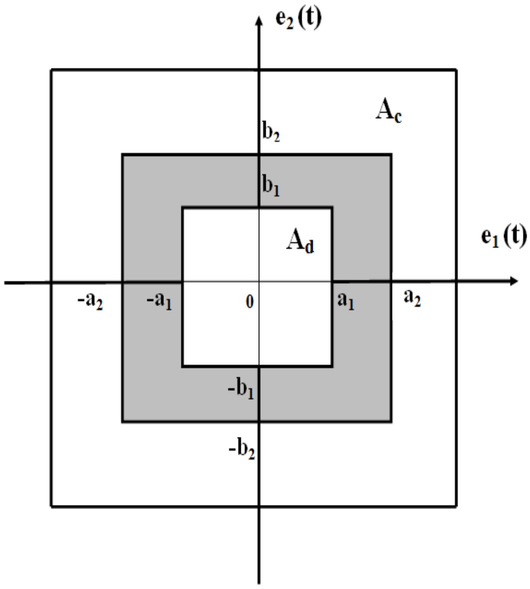

Fig. 4: Controller regions.

The initial conditions are defined as:
$x_1(0) = 10$, $x_2(0) = 1$, $y_1(0) = 5$, $y_2(0) = 11$.
Also, an assumption is made that the uncertainty term, $\Delta f(y_1, y_2) = -\sin(y_1)$ and the disturbance term, $d(t) = rand$ are bounded by $|\Delta f(y_1, y_2)| \le \alpha = 1$ and $|d(t)| \le \beta = 1$, respectively.
The simulation results are shown in Figures 5-17.
Figs. 5, 8, 11 and Fig. 14 show time series of the master and slave states corresponding to their control methods. Figs. 6, 9, 12 and Fig. 15 show time series of synchronization errors corresponding to their control methods.
Fig. 7 shows time series of LQR control effort. Figs. 10, 13 and Fig. 16 show time series of control effort and sliding surface corresponding to their control methods. Fig. 17 shows the time series of the sliding surface corresponding to hybrid control.
The simulation results of hybrid control have a good performance in comparison to other control methods that were applied in this section. The simulation results of hybrid control confirm that the master and the slave systems achieve the synchronized states before 1 sec. Also, these results demonstrate that the system error states are regulated to zero asymptotically before 1 sec.
In addition, it can be seen that the results of hybrid control have a good performance even though the overall system is subject to uncertainty and disturbance.

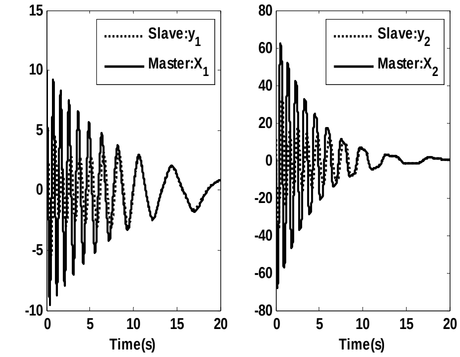

Fig. 5: Time series of the master and slave states with LQR control.

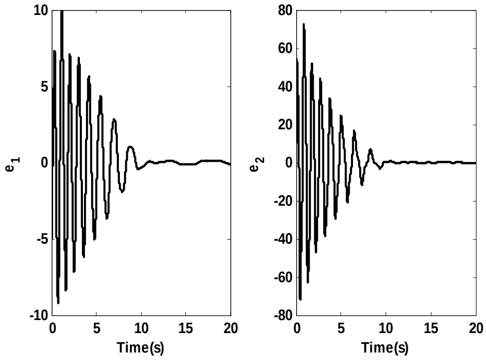

Fig. 6: Time series of synchronization errors with LQR control.

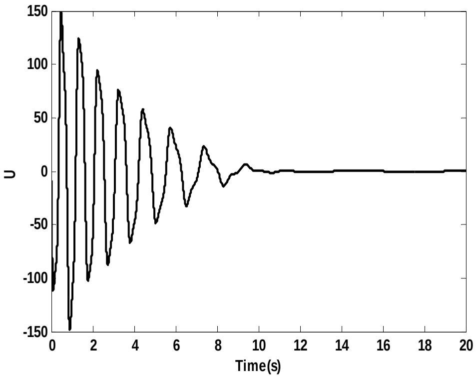

Fig. 7: Time series of LQR control effort.

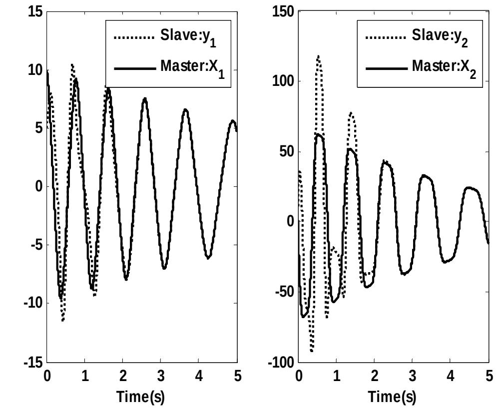

Fig. 8: Time series of the master and slave states with SM control.

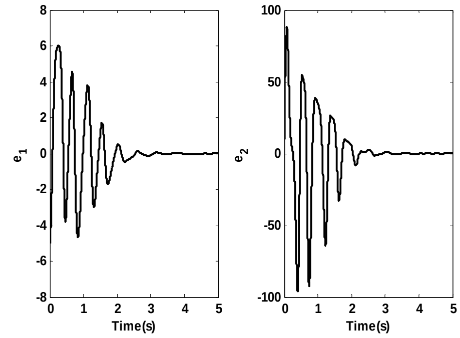

Fig. 9: Time series of synchronization errors with SM control.

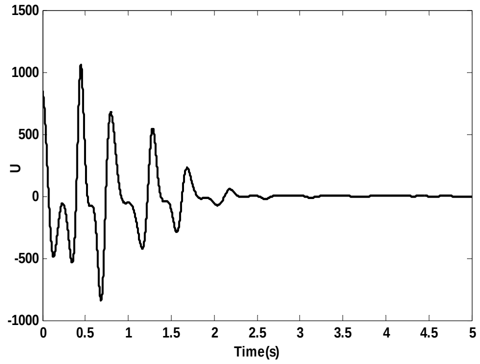

Fig. 10: Time series of SM control effort and sliding surface.

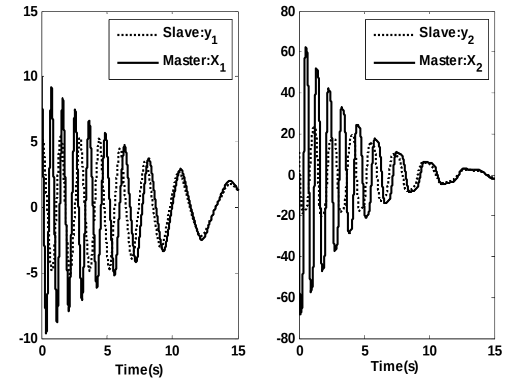

Fig. 11: Time series of the master and slave states with $GRBF_{NN}$ control.

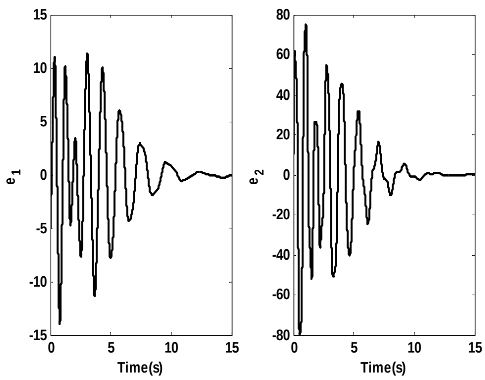

Fig. 12: Time series of synchronization errors with $GRBF_{NN}$ control.

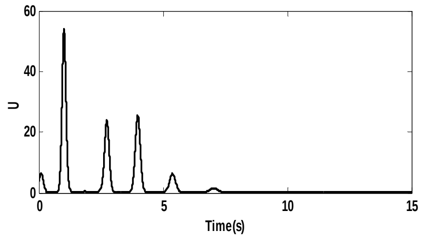

Fig. 13: Time series of $GRBF_{NN}$ control effort and sliding surface.

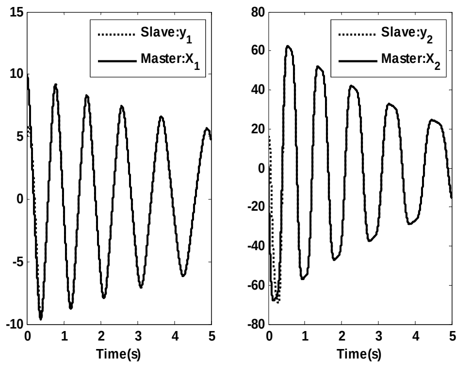

Fig. 14: Time series of the master and slave states with hybrid control.

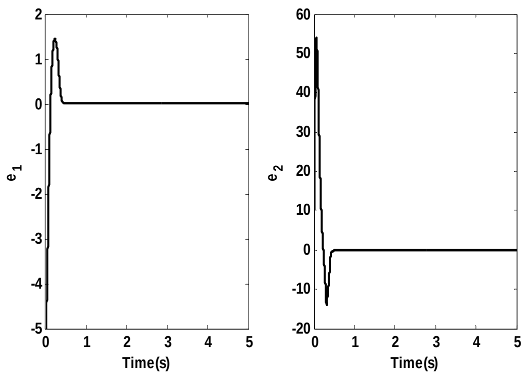

Fig. 15: Time series of synchronization errors with hybrid control.

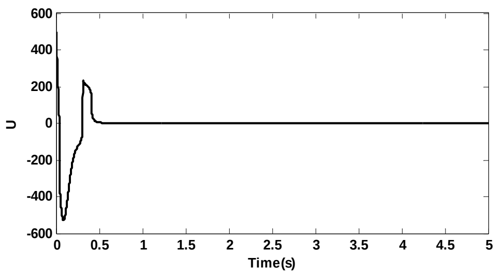

Fig. 16: Time series of hybrid control effort.

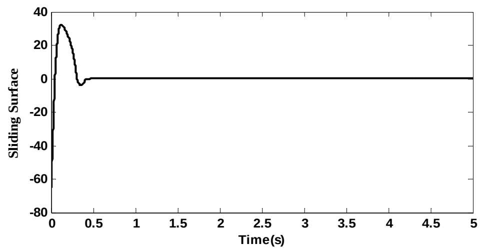

Fig. 17: Time series of sliding surface with hybrid control.

6- Conclusion

This paper presents a hybrid control scheme for the synchronization of chaotic Duffing oscillator systems characterized by system uncertainties and disturbances. This hybrid control scheme is highly robust and achieves a stable, controlled system despite the presence of uncertainties and disturbances. Simulation results of hybrid control have a good performance in comparison to other control methods applied in this paper. According to these simulations, the proposed hybrid method can be successfully applied to the synchronization problem of chaotic Duffing oscillator systems.

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