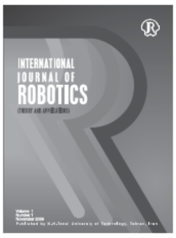


INTERNATIONAL JOURNAL OF ROBOTICS
(THEORY AND APPLICATIONS)
E-mail: IJR@kntu.ac.ir

Effect of Step Length and Step Period on Walking Speed and Energy Consumption: a Parameter Study

Mohammad Jafar Sadigh[a], Saeed Mansouri[b]

a- Department of mechanical engineering, Isfahan University of Technology, Isfahan, Iran. jafars@cc.iut.ac.ir, Postal Code: 84156-83111 IRAN, Phone: (+98) 311 3915206, (+98) 311 362 8988

b- Department of mechanical engineering, Isfahan University of Technology, Isfahan, Iran.s.mansuri@me.iut.ac.ir, Postal Code: 84156-83111 IRAN, Phone: (+98) 311 3915206, (+98) 311 362 8988

ARTICLE INFO

Keywords:
Stability, Path Planning, biped robots, zero moment point

ABSTRACT

Stability and performance are two main issues in motion of bipeds. To ensure stability of motion, a biped needs to follow specific pattern to comply with a stability criterion such as zero moment point. However, there are infinity many patterns of motion which ensure stability, so one might think of achieving better performance by choosing proper parameters of motion. Step length and step period are among major parameters through which we control our motion. Change of these parameters results in change in pattern of motion and consequently affects major characteristics of motion such as stability, speed and energy consumption. In this paper we used genetic algorithm for stable path planning for motion with different values of step length and step period. Considering actuators limit, feasible domain of motion is found. Then maximum feasible speed and consumed power is calculated and reported.

1- Introduction

Locomotion of biped robots has gained lots of attention during past few decades. While stability was the first issue which was considered by researchers, improving pattern of motion and optimization are the most important issues which are nowadays studied by researchers.

Vukobratovic et al. 0-[2] proposed a stability criterion known as zero moment point (ZMP) which guarantees stability of walking. According to them ZMP is a point on the ground at which moment of all gravitational and inertial forces acting on robot is zero about two axes that lie in the plane of the ground. They stated that a sufficient condition for stability is to keep this point within supporting polygon during motion. This method was employed by many researchers to plan stable walking gait for bipeds. Kajita et al. [3]-[4], Park and Kim [5] and Erbatur and Kurt [6] used a linear model of inverted pendulum for path planning for a stable motion of biped. However, due to approximation in dynamic model, ZMP does not exactly track the desired trajectory. Takanishi et al. [7] and Yamaguchi et al. [8] considered a dynamic model of the biped. They used Fourier series functions for hip joint and track to design the motion such that ZMP follows a prescribed trajectory. However it is difficult to determine desired ZMP trajectory to have appropriate performance. To deal with this problem, Huang et al.

[9] proposed a method of gait synthesis without first prescribing the desired ZMP trajectory. In this method foot and hip trajectories are planned beforehand in cartesian space using cubic spline interpolation. They formulated the smooth hip motion using two parameters and then derived these parameters by iterative computation with the objective is to obtain largest stability margin.

On the other hand, many researchers tried to optimize the motion. Much attention in this field is paid to minimization of consumed energy [10]-[12]. Optimization of actuator demand is another subject which is studied [13]-[16]. Dau et al. [12] planned the foot and hip trajectories in cartesian space using polynomial interpolation. The characteristics of the trajectories are governed by the seven key parameters whose values are optimized by using Genetic Algorithm (GA). Capi et al. [14] proposed a method for optimal gait generation based on GA. They considered ZMP criteria as constraint for objective function. Speed of bipeds is another important characteristic which has been improving among different generations of bipeds. Chevallereau and Aoustin[17] obtained optimal cyclic gait for a biped robot without actuated ankle. They assumed the joints variables to be polynomial functions and then find their coefficients to optimize walking speed. Dip et al. [18] optimized bipedal walking gait by considering tradeoff between stability margin and speed. They applied GA approach to optimize the key parameters of the walking trajectory and the step length for a certain step period such that stability margin is maximized. Tlalolini et al. [19] tried to design a path which could minimize energy cost of a biped while walking with a desired speed. They, then, tried to increase speed of walking by a search strategy in which they increased the desired speed step by step.Understanding the effect of change intwo basic parameters of walking; i.e. step length and step period, on maximum speed and consumed energy is the motivation behind this study, which yields feasible combinations of step length and step period for an assumed biped. To this end, a stable path generator based on work by Huang et al. [9] and using GA optimization is employed for path planning for different values of step length and step period. Horizontal position of hip joint at the beginning and end of single support phase are two parameters which are used by GA to maximize stability of biped. This procedure is repeated for 525 combinations of step length and step period. The necessary joint torques are then calculated by inverse dynamic during both single and double support phases. Considering torque limits for each joint we may construct the feasible region of motion for each joint and intersecting them, the feasible region for the parameters can be obtained, through which effect of these parameters on maximum speed and consumed energy is studied.

After this introduction, derivation of dynamic model is presented in section two. Path generation is described in the following section. Result obtained from parameter study, which shows the feasible region of step length and step period and the effect of these parameters on consumed energy are given in fourth section. This section followed by some concluding remarks in the last section

2-Dynamic Model

Considering the purpose of this study, we may use a seven-link model of a planar biped, as shown in Fig. 1, which consists of a trunk and two legs with three joints of hip, knee and ankle. It is assumed that all joints are revolute and actuated by rotational actuators co-located at the joints.

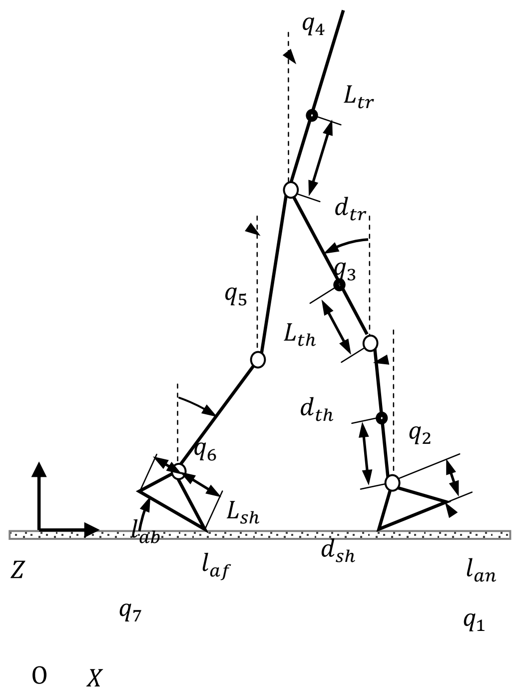


Fig.1: Seven link biped robot model

.

One complete step includes two phases of single support (SSP) and double support (DSP) see Fig. 2. During single support phase, which starts when one leg separates from the ground and ends when it again comes in contact with the ground, one leg is virtually pivoted to the ground while the other leg is swinging in forward direction. In this phase the system has open chain configuration which is described by an array of generalized coordinates of $q_1, \dots, q_7$. Equations of motion for the system during this phase can be obtained in general form as:

$$M(q)\ddot{q} + h(q,\dot{q}) = B(q) \quad (1)$$

where $M(q)$ is a 7×7 symmetric inertia matrix, $H(q,\dot{q})$ is a 7×1 array representing coriolis, gravitational and centrifugal terms, $B(q)$ is a 7×6 matrix andτ is the 6×1 vector of input torques.

Double support phase starts at the end of SSP, when the swing leg comes in contact with the ground and ends at the beginning of next SSP, when the other leg, stance leg, starts to detach from the ground. In this phase the system has two holonomic constraints defined as:

$$\varphi(q) = \begin{cases} x_h - D_s + l_{ab} = 0 \\ z_h = 0 \end{cases} \quad 0 < t \le T_{d_1}$$

$$\varphi(q) = \begin{cases} x_t - l_{af} = 0 \\ z_t = 0 \end{cases} T_{d_1} < t \le T_d \quad (2)$$

in which $(x_t, z_t)$and (x_h, z_h) are, respectively, positions of tip of rear and heel of front foot. Also, l_{ab} and l_{af} are heel and toe portions of foot, see Fig. 1, and T_{d_1} is time elapse of first portion of DSP, as shown in Fig. 2. In this phase the system has 5 DOFs. Equations of motion of the system in this phase can be obtained as:

$$M(q)\ddot{q} + H(q,\dot{q}) = B(q)\tau + J^T{}_{7\times2}\lambda_{2\times1} \quad (3)$$

inwhich M, H, B and$\tau$are defined as before andλ is array of Lagrange multipliers and J depicts Jacobian of constraints; i.e. $J = \partial\varphi/\partial q$.

To eliminate Lagrange multipliers one may premultiply (3) by transpose of orthogonal compliment of J; i.e. matrix J_c which satisfies $JJ_c = 0$. As J_c is a 5×7 matrix, the resulting equations represent a set of five new equations with seven unknown of $q_1, \dots, q_7$.

$$\bar{\bar{M}}_{5\times7}\ddot{q} + \bar{\bar{H}}_{5\times1} = \bar{\bar{B}}_{5\times6}\tau_{6\times1} \quad (4)$$

in which $\bar{\bar{M}} = J_c{}^T M$, $\bar{\bar{H}} = J_c{}^T H$ and $\bar{\bar{B}} = J_c{}^T B$.

The system in this phase is over actuated; i.e. it has five DOFs and six actuators, which means that inverse dynamics during this phase has infinity many solution.

3.Path Planning

By path planning we mean generating desired time history of joint angles such that all physical constraints of motion as well as stability criterion are satisfied. Moving joints with such desired joint angles would result in stable walking pattern. To this end, we first plan cartesian trajectories of hip and ankle joints having these trajectories, one might use inverse kinematic relations to compute joint angles. As stated in first section the method of path generation is adopted from [9] and is presented briefly for the sake of completeness.

As walking is periodic we may just plan the motion of one leg for one complete step from start of DSP to the end of SSP. Duration of DSP and SSP are two important parameters which should be planed. To achieve a good walking speed as well as good capability for stabilizing motion we may choose period of DSP to be 30% of the whole step period; i.e. $T_d = 0.3T_c$, where T_d and T_c depict periods of DSP and the whole step, [4].

a. Ankle Trajectory

It is a common practice to consider foot angle equal zero; however, in this study we choose a more realistic, condition in which foot angle at the beginning of SSP and DSP are $\theta_b$and θ_f, respectively. Assuming motion on flat surface foot angle should satisfy following constraints:

$$q_{a_r}(t) = q_7(t) = \begin{cases} 0 & 0 \le t \le T_{d_1} \\ \theta_b & t = T_d \\ \theta_f & t = T_c \\ 0 & T_c + T_{d_1} \le t \le 2T_c \end{cases} \quad (5)$$

in which q_{a_r} depicts joint angle of right ankle and θ_b and θ_f are chosen as:

$$\begin{cases} \theta_b = W_{\theta_b} D_s \\ \theta_f = W_{\theta_f} D_s \end{cases} \quad (6)$$

whereW_{θ_b} and W_{θ_f} are some constants.

To avoid impact on the heel at the beginning of DSP and to have continuous trajectory for $q_{a_r}$and ZMP the following constraints must be satisfied:

$$\begin{cases} \dot{q}_{a_r} = 0, \ddot{q}_{a_r} = 0 & t = T_{d_1} \\ \dot{q}_{a_r} = 0 & t = T_d \\ \dot{q}_{a_r} = 0 & t = T_c \\ \dot{q}_{a_r} = 0, \ddot{q}_{a_r} = 0 & t = T_c + T_{d_1} \end{cases} \quad (7)$$

To avoid collision of feet with ground the following conditions should also be satisfied:

$$x_{a_r}(t) = \begin{cases} 0 & 0 \leq t \leq T_{d_1} \\ l_{af} - l_{af} \cos q_{a_r} + l_{an} \sin q_{a_r} & T_{d_1} \leq t \leq T_d \\ L_{ao} & t = T_m \\ 2D_s - l_{ab} + l_{ab} \cos q_{a_r} + l_{an} \sin q_{a_r} & T_c \leq t \leq T_c + T_{d_1} \\ 2D_s & T_c + T_{d_1} \leq t \leq 2T_c \end{cases} \quad (8)$$

and

$$z_{a_r}(t) = \begin{cases} l_{an} & 0 \leq t \leq T_{d_1} \\ l_{af} \sin q_{a_r} + l_{an} \cos q_{a_r} & T_{d_1} \leq t \leq T_d \\ H_{ao} & t = T_m \\ l_{ab} \sin(-q_{a_r}) + l_{an} \cos q_{a_r} & T_c \leq t \leq T_c + T_{d_1} \\ l_{an} & T_c + T_{d_1} \leq t \leq 2T_c \end{cases} \quad (9)$$

Here we assumed L_{ao} to be proportional to D_s; i.e. $L_{ao} = W_{L_{ao}} D_s$.

In these relations, (L_{ao}, H_{ao}) show the position of the highest point of the swing foot as shown in Fig. 2 where T_m is the time when the right foot is at its highest position, l_{an} is the height of the foot as shown in Fig. 1. Since right foot ankle joint is at its highest position at T_m, the first time derivative of z_{a_r} at T_m must be zero. In this study weconsidered two polynomials of fifth and fourth order in terms of time for the first portion of ankle joint motion in z and x direction and two forth order polynomials for their second portion of motion. Coefficients of these polynomials are chosen such that boundary conditions defined by (5) to (9) are satisfied.

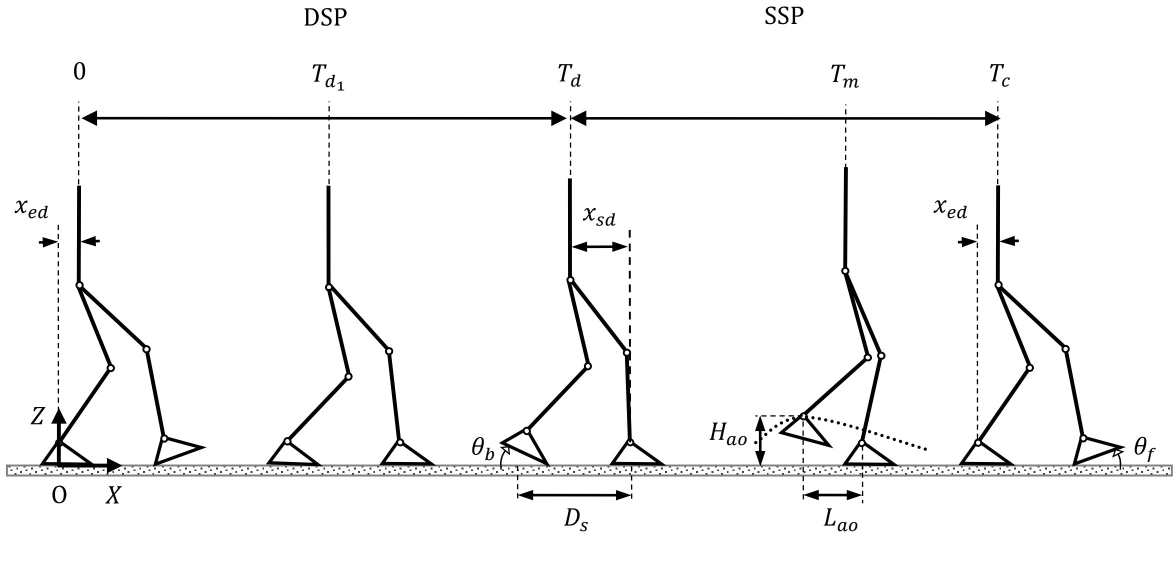


Fig.2: Motion pattern of a biped robot for a half cycle.

b. Hip Trajectory

Hip trajectory has a great impact on quality of walking, stability of motion and knee joint torque demand. To have a comfortable walking we wish to have the trunk angle to be zero; i.e. $q_4 = q_h = 0$. To ensure good stability we try to design hip joint trajectory as a function of two parameters known as x_{sd} and x_{ed}[9][9], as shown in Fig. 2. In order to have periodic and continuous motion the following constraints must be satisfied.

$$\begin{cases} \dot{x}_{hip}(t=0) = \dot{x}_{hip}(t=T_c) \\ \ddot{x}_{hip}(t=0) = \ddot{x}_{hip}(t=T_c) \end{cases}$$

Also the following geometric constraints should be satisfied:

$$x_{hip}(t) = \begin{cases} x_{ed} & t = 0 \\ D_s - x_{sd} & t = T_d \\ D_s + x_{ed} & t = T_c \end{cases}$$

In order to achieve a better energy performance the hip trajectory is considered as:

$$z_{hip}(t) = \begin{cases} H_{min} & t = T_{d_1} \\ H_{max} & t = T_m \end{cases} \tag{12}$$

Which means hip joint is at its highest position at time T_m and it is at its lowest position when both feet are completely on the ground. These factors are chosen as:

$$\begin{cases} H_{min} = \sqrt{(l_{sh} + l_{th})^2 - 0.25{D_s}^2} + l_{an} - \epsilon \\ H_{max} = H_{min} + \delta \end{cases}$$

whereϵ and δ are two arbitrary factors.

Here we used two third order polynomials in terms of time to describe horizontal and vertical position of hip joint. Coefficients of these polynomials are chosen such that (10) to (13) are satisfied.

c. Stability Criterion

To ensure stability of motion we use ZMP criterion which was first introduced by Vukobratovic[2]. ZMP is defined as the point on ground where the sum of all moments acting on the system, due to gravity and inertia forces about that vanishes. If ZMP is located within the supporting polygon, the motion is stable. Position of ZMP on the ground can be computed using the following equation [9]:

$$x_{zmp} = \frac{\sum_{i=1}^{n} m_i(\ddot{z}_i+g)x_i - \sum_{i=1}^{n} m_i\ddot{x}_i z_i - \sum_{i=1}^{n} I_{yi}\ddot{q}_i}{\sum_{i=1}^{n} m_i(\ddot{z}_i+g)} \tag{14}$$

Stable region for considered pattern of motion is shown in Fig. 3, in which solid lines show the boundary of stable region and the dotted line shows the position of ZMP for best stability margin.

In this study we first choose a set of suitable parameters of walking; i.e. T_{d_1}, T_{d_2}, T_d, T_m, W_{θ_b}, W_{θ_f}, $W_{L_{ab}}$, ε, δ, T_c and D_s, as introduced in previous section. Then we employed GA to find the best values of x_{sd} and x_{ed} to optimize the cost function J which minimizes deviation of actual ZMP trajectory, x_{zmp}, from its desired path, x^*_{zmp}, shown by dotted line in Fig. 3. Stability index is defined as:

$$J = \begin{cases} max(\delta^+, \delta^-), & \delta^+ > 0 \; or \; \delta^- > 0 \\ -\frac{1}{\int_0^{T_c} |x_{zmp} - x^*_{zmp}| dt}, & \delta^+ \le 0 \; and \; \delta^- \le 0 \end{cases} \tag{15}$$

in which $\delta^+$and δ^-show deviation of ZMP from its maximum and minimum acceptable values defined as:

$$\delta^+ = max\,(x_{zmp} - U), \quad \delta^- = max\,(L - x_{zmp})$$

in these relations, L and U are lower and upper boundaries of stable region.

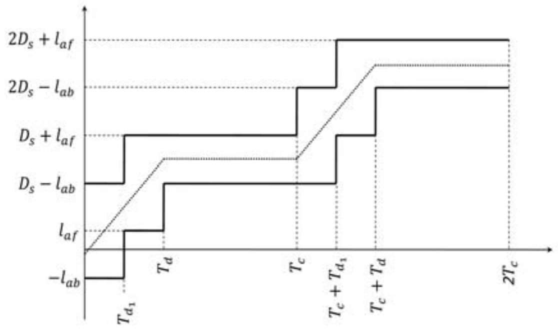


Fig.3: Stable region in a walking step.

The flow chart of GA is shown in Fig. 4. As explained before, parameters x_{sd} and x_{ed} will be found by GA in order to find the best trajectory that minimizes J. At the first step of the process, an initial population is created as a starting point. With a trajectory based on the initial parameters, the objective function is evaluated. To minimize the objective function the GA operators – selection, crossover and mutation – regenerate chromosomes. This process is repeated until the number of generations reaches the maximum number of generations or if the value of the objective function does not change for determined conservative generations. In this paper we use GA toolbox of MATLAB 2010 with parameters listed in table 1.

Table.1:Parameters of GA

Parameters	Values
Chromosome length	2
Population size	20
Initial population	[0.1,0.05]
Crossover ratio	0.8

4-Feasible Region of Step length and Step Period

In this section we use the path generator and the developed equations of motion presented in previous sections to investigate admissible domain of twomajor parameters of walking, namely step length and step period. These two parameters play central role in moving biped with different speeds. In another words, if a legged creature wants to increase its speed

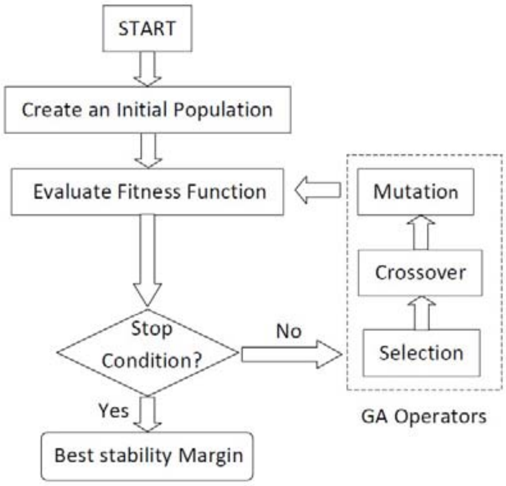
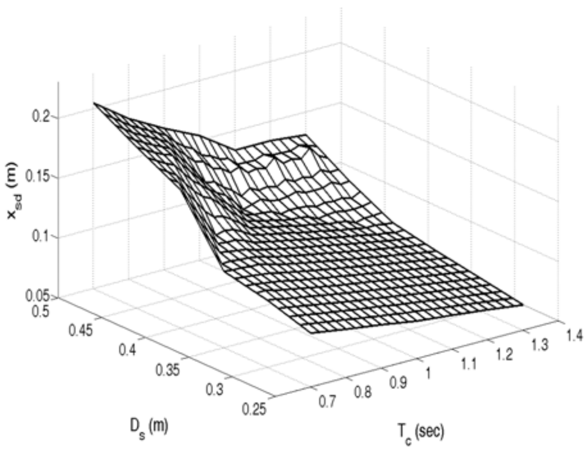


Fig.4: Flow chart of the proposed GA .

of motion, it either should increase the step length or decrease the step period, or even both at the same time. Now the question is what is the effect of these decisions and to what limit each of them could be pushed. Moreover, it might be interesting to investigate what would be the effect of such decision on stability margin and energy consumption.

To understand this we arranged a parameter study in which considering physical characteristics of the biped, given in table. 2, we choose lower and upper feasible boundaries of step length and step period as $D_{s_{min}} = 0.25$ m, $T_{c_{min}} = 0.7$ s, $D_{s_{max}} = 0.5$ m and $T_{c_{max}} = 1.3$ s. Then a stable path is generated using described GA for 21 values of D_s and 25 values of T_c. Fig. 5 shows the values of input parameters $x_{sd}$and x_{ed}, see Fig. 5, which minimizes the cost function J.



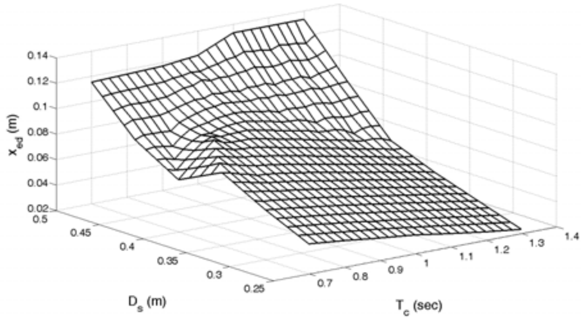


Fig.5: Values of x_{sd}andx_{ed}.

Once the path is known the necessary torque in each joint is computed using inverse dynamic equations given by Eq. (1) and Eq. (4) for SSP and DSP. Fig. 6 and Fig. 7 show typical result for the generated stable path and joint torques corresponding to $T_c = 1$ secand $D_s = 0.35$ m.

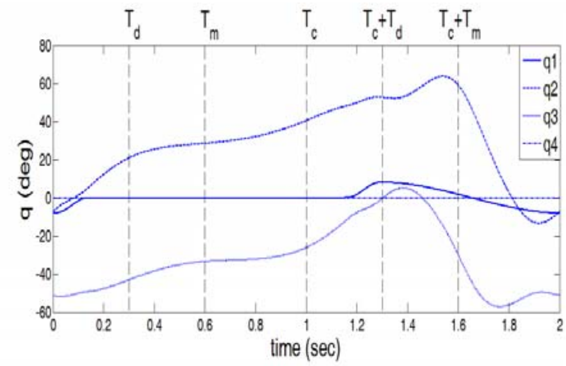


Fig. 6: Time history of joint angles during one complete step.

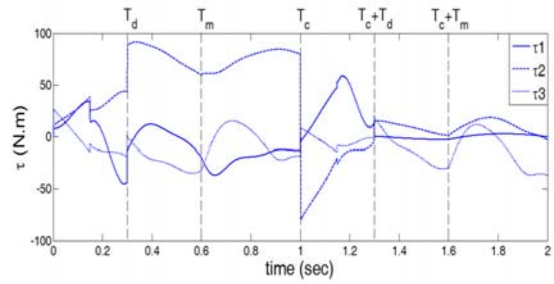


Fig.7: Time history of joint torques during one complete step.

We define a point (D_s, T_c) feasible if all necessary joint torques are within the corresponding actuator limits. In order to find the feasible region we repeated above calculations for 525 points; i.e. 21 D_s and 25 T_c. Fig. 8 shows maximum and minimum values of necessary torques for left hip joint for different values of (D_s, T_c). In this figure each branch corresponds to a specific value of T_c. Any point (D_s, T_c) shown in these graphs which can produce a stable motion and is located below $\tau_{\max}$ and above τ_{min} is considered admissible. Clearly the points located on the τ_{max} andτ_{min} lines represent the boundary of admissible area.

The study shows that for each specific step period there exist a critical step length above which no stable motion is possible. So the feasible region is the intersection of stable region and admissible region in $T_c - D_s$ plane, which is shown by dotted line.

As the motion is periodic, computation is done for each leg in half cycle. So for each joint we will have two graphs, one for right leg in first half cycle and the other for left leg in next half cycle. A feasible set of (D_s, T_c) for each joint is one that is feasible in both half cycles; this means that feasible area in $T_c - D_s$ plane for each joint is the intersection of two feasible areas corresponding to both legs. Fig. 9 shows feasible areas for all restrictive joints of both right and left legs.

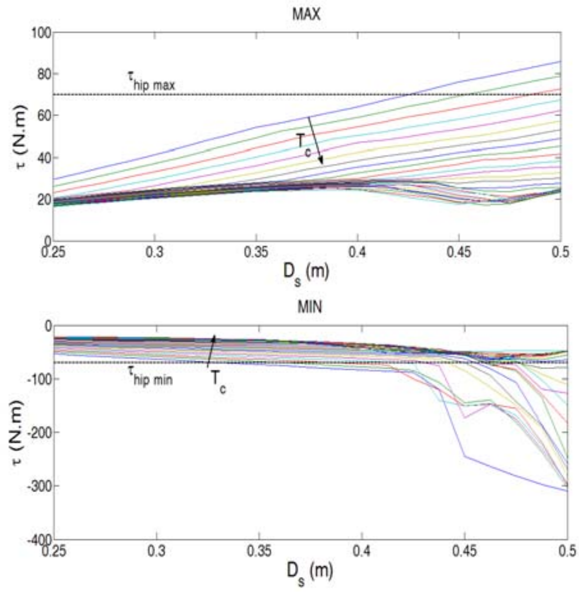
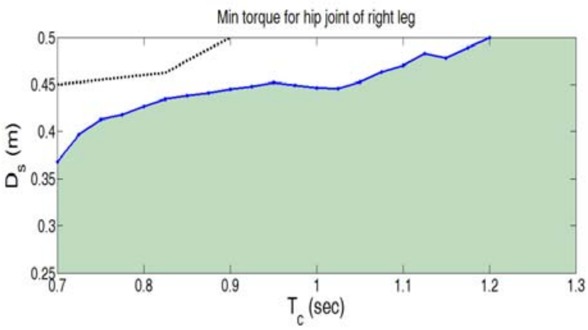


Fig. 8:Maximum and minimum torque for hip joint of Left leg.



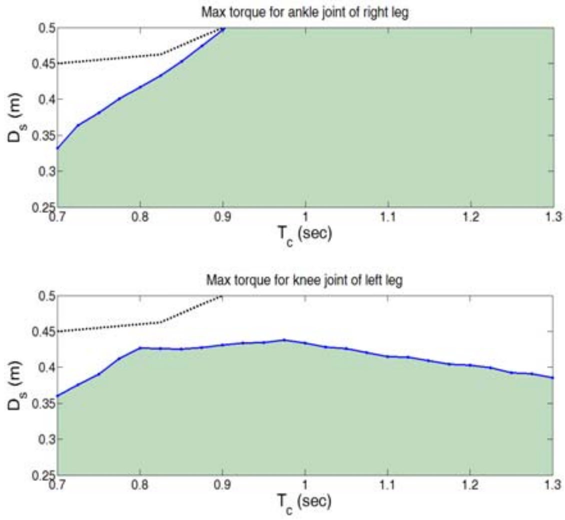


Fig. 9: Feasible region, shaded area, due to most restrictive situation in each joint.

One can see from Fig. 9 that with assumed physical parameters of biped, right knee imposes no restriction. We may also learn from Fig. 9 that performance of such biped can be improved by increasing max torque of actuator of knee joint and minimum limit of actuators of hip and ankle joints.

Apart from such information needed for design purpose, we may use the results to find out about maximum speed that this biped can achieve. To this end, we may find the feasible area, shown in Fig. 10, intersecting all of the feasible areas given in Fig. 9. Now for any feasible point the average speed would be D_s/T_c, which shows the slope of a line connecting origin to the point (T_c, D_s). We may also map this feasible area to the plane of (T_c, v), as shown in Fig. 11. This figure shows what would be the feasible values of velocity for each value of T_c.

By inspection from Fig. 10 or Fig. 11 it can be seen that this biped can achieve maximum velocity of 0.524 (m/s) by a step length of 0.426 (m) and stepperiod of 0.814 (s). This point $({D_s}^*, {T_c}^*)$ corresponds to the values of 0.1441 and 0.0825 for x_{sd} and x_{ed} respectively. Fig. 12 shows the trajectoryof ZMP for

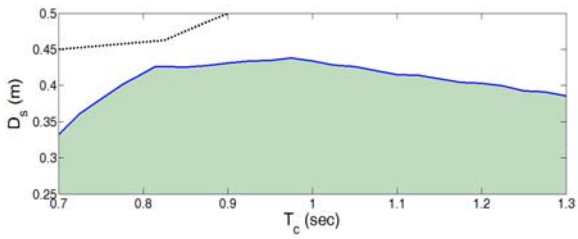


Fig. 10: Admissible region of step length and step period for biped robot.

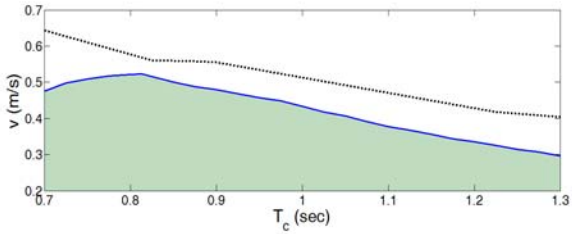


Fig. 11: Admissible region of step period and speed for biped robot.

this point, which lies within stable region. Schematic of motion of biped in motion with such D_s and T_c is shown in Fig. 13.

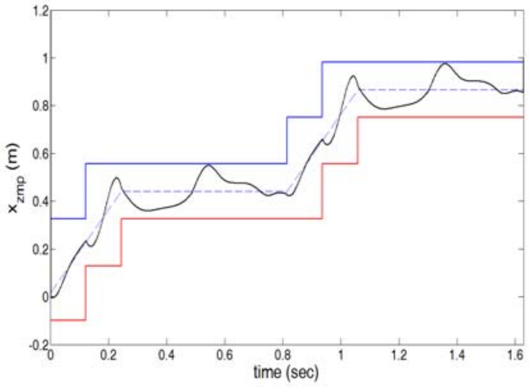


Fig. 12: ZMP trajectory and stable region.

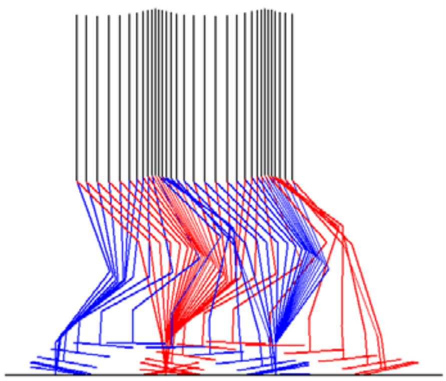


Fig. 13: Schematic of biped motion.

Energy consumption is another important factor in biped walking. In this study, we also computed the power consumption for each combination of step length and step period. Fig. 14 show meanpower consumption as a function of step period and step length. The study reveals that for each velocity there is specific value of step length for which power consumption is minimum. More over these figures show that increasing step length with constant velocity results in better energy performance.

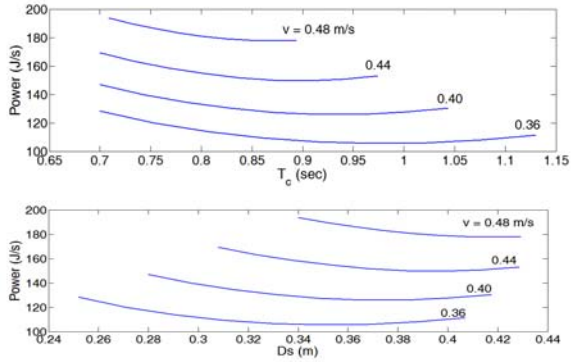


Fig. 14:Power consumption as a function of step period, and step length.

Table.1:The Physical Parameters of the Biped Robot.

Mass (kg)	m_{foot}	m_{shin}	m_{thigh}	m_{trunk}
	3.3	5.7	10	30
Inertia $(\mathrm{kg.m^2})$	$I_{y_{foot}}$	$I_{y_{shin}}$	$I_{y_{thigh}}$	$I_{y_{trunk}}$
	0.01	0.02	0.08	1.7
Length (m)	l_{sh}	l_{th}	d_{sh}	d_{th}
	0.3	0.3	0.15	0.15
	d_{tr}	l_{an}	l_{ab}	l_{af}
	0.25	0.1	0.1	0.13
Nominal Torques (N.m)	$\lvert\tau_{hip}\rvert$	$\lvert\tau_{knee}\rvert$	$\lvert\tau_{ankle}\rvert$	
	≤ 70	≤ 100	≤ 70	

Table.2:The Physical Parameters of the Biped Robot.

Time (sec)	T_{d_1}	T_{d_2}	T_d	T_m	
	$0.15T_d$	$0.15T_d$	$0.3T_c$	$0.5T_c$	
Weigh t	W_{θ_b}	W_{θ_f}	$W_{L_{ao}}$	ϵ	δ
	$0.4\frac{rad}{m}$	$-0.4\,\frac{rad}{m}$	0.63	$0.08\,m$	$0.02\,m$

5-Conclusion

In this paper effect of change in step length and step period of motion for bipeds is studied. Considering saturation limit for actuators, feasible area of stable motion of biped in plane of step length-step period parameters are obtained by parameter study among all stable possible paths for a specific biped. The results which could be considered as main contribution of this paper reveal some interesting results as follows:

1- For each value of step period there exists a maximum feasible step length. Decreasing step period beyond certain value results in decrease of this maximum feasible step length.

2- Knee and hip joints are more sensitive to decreasing step period. In other words, increasing step frequency beyond certain value might cause hip and knee joint actuators to saturate.
3- For a certain velocity there exist a step length/step period for which power consumption is minimum.
4- For a certain velocity increase of step length results in decrease of power consumption at all small step length.

The study helped to construct feasible area of motion in $T_c - D_s$ and $T_c - v$ planes which can be used to establish a robust pattern of motion as for as these parameters are concerned.

References

[1] M. Vukobratovic and J. Stepanenko, "On the stability of anthropomorphic systems," Mathematical Biosciences, vol. 15, pp. 1-37, 1972.
[2] M. Vukobratovic and B. Borovac, "Zero-moment point—thirty five years of its life," International Journal of Humanoid Robotics, vol. 1, no. 1, pp. 157-173, 2004.
[3] S. Kajita and K. Tani, "Study of dynamic biped locomotion on rugged terrain—derivation and application of the linear inverted pendulum mode," Proceedings of the IEEE International Conference on Robotics and Automation, pp. 1405-1411, 1991.
[4] S. Kajita, F. Kanehiro, K. Kaneko, K. Yokoi and H. Hirukawa, "The 3D linear inverted pendulum mode: a simple modeling for a biped walking pattern generation," Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 239-246, 2001.
[5] J.H. Park and K.D Kim, "Biped robot walking using gravity-compensated inverted pendulum mode and computed torque control," Proceedings of IEEE International Conference on Robotics and Automation, pp. 3528-3533, 1998.
[6] K. Erbatur and O. Kurt, "Natural ZMP trajectories for biped robot reference generation," IEEE Transactions on Industrial Electronics, vol. 56, no. 3, pp. 835-845, 2009.
[7] A. Takanishi, M. Tochizawa, H. Karkai and I. Kato, "Dynamic biped walking stabilized with optimal trunk and waist motion," Proceedings of IEEE/RSJ International Workshop on Intelligent Robots and Systems, pp. 187-192, 1989.
[8] J.I. Yamaguchi, A. Takanishi and I. Kato, "Development of biped walking robot compensating for three-axis moment by trunk motion," Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 561-566, 1993.
[9] Q. Huang, K. Yokoi, S. Kajita, K. Kaneko, H. Arai, N. Koyachi and K. Tanie, "Planning walking patterns for a biped robot," IEEE Transactions on Roboticsand Automation, vol. 17, no. 3, pp. 280-289, 2001.
[10] P.H. Channon, S.H. Hopkins and D.T. Pham, "Derivation of optimal walking motions for a bipedal walking robot," Robotica, vol. 10, pp. 165-172, 1992.
[11] J.H. Park and M.S. Choi, "Generation of an optimal gait trajectory for biped robots using a genetic algorithm," JSME International Journal, Series C, vol. 47, no. 2, pp. 715-721, 2004.
[12] V.H. Dau, C.M. Chew and A.N. Poo, "Achieving energy-efficient bipedal walking trajectory through GA-based optimization of key parameters," International Journal of Humanoid Robotics, vol. 6, no. 4, pp. 609-629, 2009.
[13] M. Rostami and G. Bessonet, "Sagittal gait of a biped robot during the single support phase. Part 2: optimal motion," Robotica, vol. 19, pp. 241-253, 2001.
[14] G. Capi, Y. Nasu, L. Barolli, K. Mitobe and K. Takeda, "Application of genetic algorithms for biped gait synthesis optimization during walking and going up-stairs," Advanced Robotics, vol. 15, no. 6, pp. 675-694, 2001.
[15] T. Saidouni and G. Bessonnet, "Generating globally optimised sagittal gait cycles of a biped robot," Robotica, vol. 21, pp. 199-210, 2003.
[16] G. Bessonnet, S. Chesse and P. Sardain, "Optimal gait synthesis of a seven-link planar biped," International Journal of Robotics Research, vol. 23, no. 10-11, pp. 1059-1073, 2004.
[17] C. Chevallereau and Y. Aoustin, "Optimal reference trajectories for walking and running of a biped robot," Robotica, vol. 19, pp. 557-569, 2001.
[18] G. Dip, V. Prahlad and P.D. Kien, "Genetic algorithm-based optimal bipedal walking gait synthesis considering tradeoff between stability margin and speed," Robotica, vol. 27, pp. 355-365, 2009.
[19] D. Tlalolini, C. Chevallereau and Y. Aoustin, "Comparison of different gaits with rotation of the feet for a planar biped," Journal of Robotics and Autonomous Systems, vol. 57, no. 4, pp. 371-383, 2009.
[20] Inman, V.T., Ralston, H.J., Todd, F., and Lieberman, J.C., "Human Walking," Baltimore, MD: Williams & Wilkins, 1981.