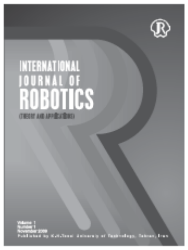

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An Efficient Algorithm for Workspace Generation of Delta Robot

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ABSTRACT

Dimensional synthesis of a parallel robot may be the initial stage of its design process, which is usually carried out based on a required workspace. Since optimization of the links lengths of the robot for the workspace is usually done, the workspace computation process must be run numerous times. Hence, importance of the efficiency of the algorithm and the CPU time of the workspace computation are highlighted. This article exerts an improved numerical search method for workspace generation of a Delta robot. The algorithm is based on a methodology applied to a Hexapod manipulator somewhere else, while the improvement utilized here causes a good increase in its speed and efficiency. The results illustrate that the approach is feasible, practical, and more efficient than initial method for the generation and analysis of the workspace of the parallel manipulator, however it is done for a Delta here.

1. Introduction

Parallel manipulators are more advantageous over the open serial robots. Some of the benefits are that they have good stiffness and high speed. Between parallel robots, whose Degree Of Freedom (DOF) is less than six received more attention for special usages, because their control is simple and their costs are low. Delta robot is Clavel's Pioneer work in the early 1990's, which is categorized as a parallel robot with three translational DOF [1]. Fig. 1 presents the manipulator consisting of a fixed and a moving platform, with three legs. The platform is connected with each drive by two links forming a parallelogram. By the special architecture, the platform is allowed to have just translational movement being parallel to the base plate. Due to the particular structure of the robot, it is able to have fast translational motion in 3-dimensional Cartesian-space. Hence, the manipulator is principally suitable for pick and place task.

The manipulator has obtained a lot of researchers' interests; however, Clavel himself completed the kinematic and dynamic analysis of the manipulator [2]. Formerly, many researchers focused on the generation of the workspace. Various approaches have been utilized to determine the workspace of a parallel robot, such as discretization method, geometrical method, and analytical method. For the discretization method, a grid of nodes with their position and orientation is defined. Then, the kinematic analysis is applied for each node. The approach is straight forward and its strategy is to verify whether the kinematic analysis is solvable for each node or no, and although to make sure whether joint limit reaches or link interference occurs [3,4].

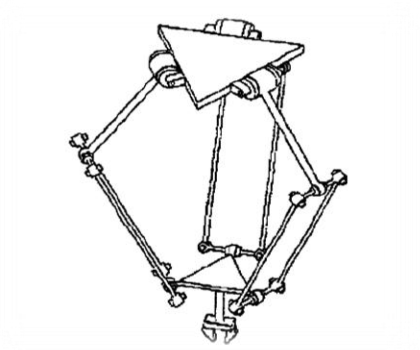

Figure 1. A schematic representation of Delta manipulator [2].

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For the discretization method, a grid of nodes with their position and orientation is defined. Then, the kinematic analysis is applied for each node. The approach is straight forward and its strategy is to verify whether the kinematic analysis is solvable for each node or no, and although to make sure whether joint limit reaches or link interference occurs [3, 4]. The discretization method is simple to implement, but it has some serious shortcomings.

It is expensive in computation and its results are limited to the nodes of the grid. Thus, there is no guarantee that the space between two valid nodes is acceptable, too. Using geometrical method, the workspace can be calculated as intersection of simple geometrical objects [5, 14]. The main interest of the geometrical approach is that it is usually very fast and accurate but it needs a good computational geometry library to perform the calculations. Analytical method is more complicated but be able to distinguish between multiple solutions easily. Conti et al. also proposed a numerical method to evaluate the workspace variation of a Hexapod machine tool using spherical coordinates [6]. The main contribution of this work is to propose an improved numerical algorithm to generate workspace boundary of Delta robot based on the methodology employed by Conti et al. for a Hexapod manipulator. The results show that the applied improvements increase the efficiency satisfactorily.

2. Kinematic Analysis

Since the Delta manipulator includes several closed loops, it is extremely complicated to analyze its kinematics. To simplify the model and to reduce the number of parameters, the following assumptions are made [7]:

- The moving platform always stays parallel to the base plate and its orientation about the axis perpendicular to the base plate is constantly zero. Thus, the parallelogram type joints (forearm) can be substituted by simple rods without changing the robot kinematic behavior.
- The revolute joints (one between the base plate and the upper arms, and one between the forearms and the moving platform) are identically placed on a circle, as shown in Fig. 2, thus, the moving platform can be replaced by a point P, where the three forearms are connected together.

2.1. Forward Kinematic Description

The forward kinematics, which also called the direct kinematics of a parallel manipulator, determines the x, y and z positions of the moving platform in the base-frame for any given configuration of the robot in form of angles θ1j $(j = 1,2,3)$ of the actuated revolute joints.

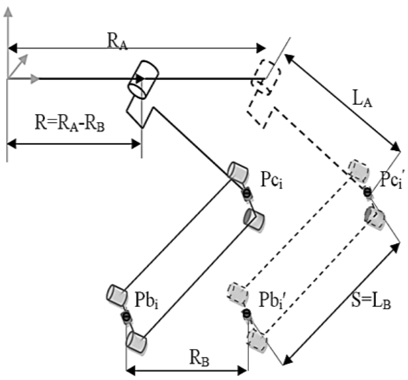


Figure 1. Model simplification of the Delta robot.

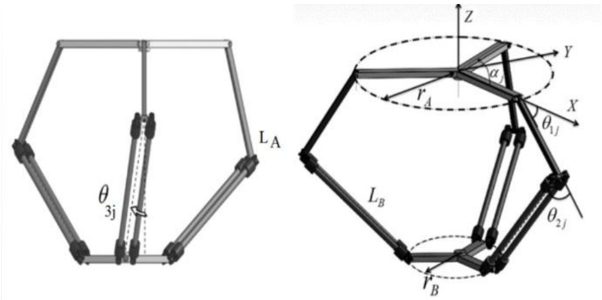


Figure 2. The parameters of Delta robot.

The geometric parameters of the Delta robot are, L_A, L_B, r_A, r_B and θ1j as defined in Fig. 3, and its variables are θ_{ij} $(i, j = 1,2,3)$, the joint angles defining the configuration of each link. If P is a point located on the moving platform, the geometric relations can be written as [8]:

$$\boldsymbol{X_p} = \mathrm{Cos}\,\alpha_j\,(r_A + L_A \mathrm{Cos}\,\theta_{1j} + L_B \mathrm{Cos}\,\theta_{3j} \mathrm{Cos}(\theta_{1j} + \theta_{2j}) - r_B) - L_A \mathrm{Sin}\,\alpha_j \mathrm{Sin}\,\theta_{3j} \tag{1}$$

$$\boldsymbol{Y_p} = \mathrm{Sin}\,\alpha_j\,(r_A + L_A \mathrm{Cos}\,\theta_{1j} + L_B \mathrm{Cos}\,\theta_{3j} \mathrm{Cos}(\theta_{1j} + \theta_{2j}) - r_B) + L_B \mathrm{Cos}\,\alpha_j \mathrm{Sin}\,\theta_{3j} \tag{2}$$

$$\boldsymbol{Z_p} = L_A \mathrm{Sin}\,\theta_{1j} + L_B \mathrm{Cos}\,\theta_{3j} \mathrm{Sin}(\theta_{1j} + \theta_{2j}) \tag{3}$$

$[X_p, Y_P, Z_p]$ Are the coordinates of point P in the fixed or reference frame (XYZ). In order to eliminate the passive joint variables, (1) , (2) and (3) are squared and added to achieve:

$$(\boldsymbol{X_j} - \boldsymbol{X_p})^{\boldsymbol{2}} + (\boldsymbol{Y_j} - \boldsymbol{Y_p})^{\boldsymbol{2}} + (\boldsymbol{Z_j} - \boldsymbol{Z_p})^{\boldsymbol{2}} - L_B^2 = \boldsymbol{0} \tag{4}$$

where

$$\boldsymbol{X_j} = \left(r_A + L_A \operatorname{Cos}\theta_{1j}\right)\operatorname{Cos}\alpha_j\,,$$
$$\boldsymbol{Y_j} = \left(r_A + L_A \operatorname{Cos}\theta_{1j}\right)\operatorname{Sin}\alpha_j\,,$$
$$\boldsymbol{z_j} = -L_A \operatorname{Sin}\theta_{1j}\ \ ,j = 1,2,3\ and\ r = r_A - r_B.$$

Equation (4) represents three spheres with centers (X_j, Y_j, Z_j) for j=1, 2, 3 and radius LB. There are two intersection points for the three spheres, which are two solutions for forward kinematics problem of Delta robot. Solving (4) for (X_p, Y_p, Z_p) in terms of known actuators joints angles θ1j $(j = 1,2,3)$ and geometrical parameters L_A, L_B, r leads to two solutions (see Figure 4). These two solutions are located upside and downside of the robot base plate. Depending on which side of the base plate the workspace is placed, the desired solution is selected.

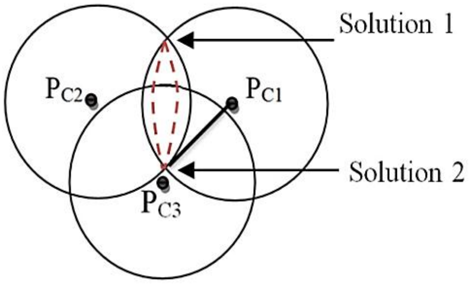


Figure 3. Two solutions of Forward Kinematics problem of Delta Robot.

2.2. Inverse Kinematic Description

Inverse kinematics problem is defined by (1), (2) and (3) that the unknowns are the rotations of actuated joints, i.e. θ_{ij} $(i, j = 1,2,3)$. The aim of this problem is to find the position of point P, namely, $\left[X_p, Y_P, Z_p\right]$. Equation (4) has been derived for the j^{th} chain, and it can be expressed as a function of Cosθ1j and Sinθ1j as follows:

$$\mathbf{M_{1j}}\operatorname{Cos}\theta_{1j} + \mathbf{M_{2j}}\operatorname{Sin}\theta_{1j} = \mathbf{M_{3j}} \qquad (5)$$

where:

$$\mathbf{M_{1j}} = -2L_A \operatorname{Cos}\alpha_j X + 2L_A - 2L_A \operatorname{Sin}\alpha_j Y,$$
$$\mathbf{M_{2j}} = 2\mathrm{L_A Z},$$
$$\boldsymbol{M_{3j}} = (L_B)^2 - (L_A)^2 - \left(X - r\operatorname{Cos}\alpha_j\right)^2 - \left(Y - r\operatorname{Sin}\alpha_j\right)^2 - Z^2 \qquad (6)$$

If $\boldsymbol{t_i} = \tan\left(\theta_{1j}/2\right)$ for i, j = 1, 2 and (3), (5) can be rewritten as:

$$\boldsymbol{t_i} = \left(\boldsymbol{M_{1j}} \pm \left[\boldsymbol{M_{1j}^2} + \boldsymbol{M_{2j}^2} - \boldsymbol{M_{3j}^2}\right]^{1/2}\right) / \left(\boldsymbol{M_{1j}} + \boldsymbol{M_{3j}}\right) \qquad (7)$$

The joint angles can be found as:

$$\boldsymbol{\theta_{1_{j,i}}} = 2\mathrm{ATAN}(\boldsymbol{t_i}) \qquad (8)$$

Equation (8) shows two possible solutions. As it was already mentioned, depending on the side of the base plate, which the workspace is placed, the solution is selected. Equations (5) are solvable if and only if:

$$\left|\left(\boldsymbol{M_{3j}}/\left(\boldsymbol{M_{1j}} + \boldsymbol{M_{2j}}\right)^{\boldsymbol{1/2}}\right)\right| \leq \boldsymbol{1} \leftrightarrow$$
$$\boldsymbol{M_{3j}^2} + \left[\boldsymbol{M_{1j}^2} - \boldsymbol{M_{2j}^2}\right] < \boldsymbol{0} \qquad (9)$$

3. The Workspace of Delta Robot

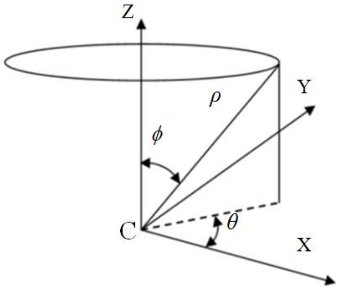
The workspace of the Delta robot is defined as a region, where can be attained by point P on the platform. To verify this property of any point in space, (9) coming from inverse kinematics, rewritten here as:

$$\boldsymbol{h_j}\left(\boldsymbol{X_p}, \boldsymbol{Y_p}, \boldsymbol{Z_p}\right) = \boldsymbol{M_{3j}^2} - \left(\boldsymbol{M_{1j}^2} + \boldsymbol{M_{2j}^2}\right) \leq \boldsymbol{0} \qquad (10)$$

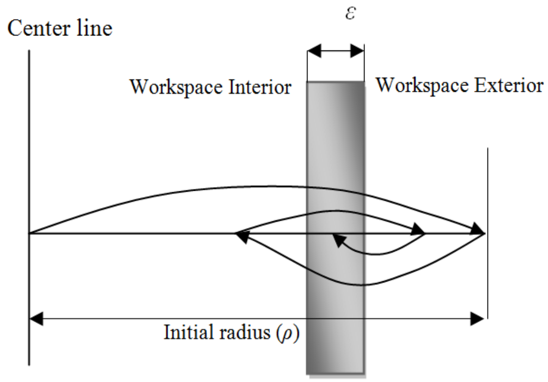
Where j=1, 2, 3.

Now, the discussion on (10) is given as follows:

- If P is inside the workspace then $h_j(P) < 0$.
- If P is on the boundary, then $h_j(P) = 0$.
- If P is outside the workspace then $h_j(P) > 0$.



b) Spherical Coordinates



a) Search Algorithm

Figure 4. Workspace search vectors (a, b) [9].

3.1. Workspace Boundaries Generation

Workspace boundaries generation is the initial stage of the robot design process. To determine the workspace boundaries of a Delta manipulator, a numerical search method, which was initially applied to a Hexapod by Conti et al., is utilized here first. In this algorithm, spherical coordinates are employed to find the boundaries of the workspace. First, a central point, namely C, is considered, then for any set of φ and θ, the radius of the point on the boundary, i.e., ρ, is found. By varying φ, θ, the whole boundary of the workspace is found. Fig. 5 graphically illustrates the methodology. The main steps of the algorithm are as follows:

1. Define the constraints and then a function, which is called Test-Pose, and returns the evaluation result, either True or False.
2. Guess an approximate center for the workspace, i.e., *C*.
3. Implement an error criterion to determine the workspace boundary. We assume an acceptable error, ε and the algorithm searches for the workspace boundary within a tolerance $\varepsilon/2$, the final value of ρ preserved.
4. Define an initial value for ρ to start the algorithm. The algorithm calculates the workspace boundary by obtaining value of radius ρ with the tolerance $\varepsilon/2$, for any $\phi \in [0,180], \theta \in [0, 360]$.

At the beginning of each step, ρ is divided by 2. If Test-Pose function for the new value of ρ returned true, this ρ added to previous, else it is subtracted.
The algorithm continues reducing the increment (Δ) to $\rho/2$, $\rho/4$, $\rho/8$ … until it is smaller than or equal to $\varepsilon/2$ and the Test-Pose function returns true.

The workspace boundary of the Delta robot is generated and visualized in MATLAB environment. To evaluate the methodology, design parameters employed in [8] are utilized. The design parameters of Delta robot are as Table 1. and Figure 6, shows the workspace boundary of the Delta robot with the design parameters according to Table 1.

Table 1. Design parameters of a Delta robot [8].

The parameters of Delta robot	L_A	L_B	R
Unit: meter (m)	1.73	2.66	1.74

3.2. Improvement of the methodology

In the above methodology, the search procedure of every iteration is started by an initial guess for boundary radius, ρ_0, and will continue until the termination condition (i.e., $\Delta > \varepsilon/2$) is fulfilled. Since, finding the radius the boundary repeats over whole interval $\varphi \in [0,180]$, and $\theta \in [0,360]$ with a constant initial radius ρ_0, it obviously involves unnecessary computations (as shown in Fig. 7).

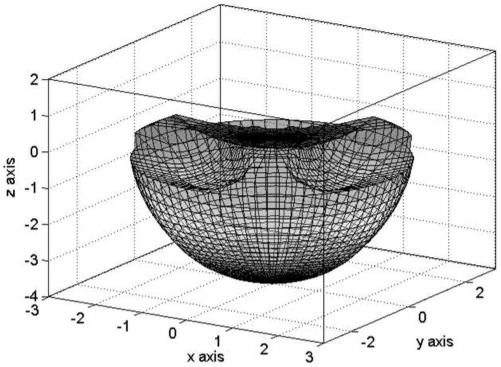


Figure 5. Workspace boundary of the Delta Robot

To improve the efficiency of the algorithm at each step over the interval $\phi \in [0,180]$, and $\theta \in [0,360]$ the following steps are added:

1. In the first stage, the same algorithm is utilized to obtain the first point on the boundary.
2. Next, the final value for ρ at each step is selected as initial value of next step, i.e., ρ_0. Compute Test-pose function for new ρ_0, θ, and φ. If the function returns True, then check whether the function is true for $\rho_0+\varepsilon$, θ, φ or not. If yes, the procedure is continued, otherwise stop and choose the last value as the radius of workspace boundary for the θ, and φ. If the function returns False, then check whether the function is true for $\rho_0-\varepsilon$, θ, φ or not. If yes, continue the procedure, otherwise, stop and choose the last value as the radius of workspace boundary for the θ, and ϕ .

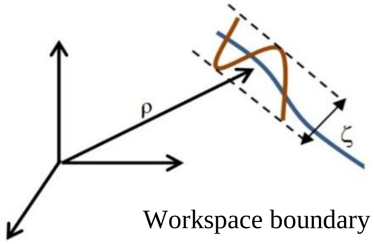


Figure 6. Workspace boundary search idea.

The radius for the next step, i.e., after considering angle increment $\Delta\phi$, a rectangular triangle is created by the last ρ, called ρ', and a tangent to the boundary with the length of $\rho' \times \Delta\phi$ as predicted in Fig. 8. The hypotenuse of the triangle is the new estimation for the current stage. Hence, Set ρ_0 at each step as

In this stage, the workspace boundary of the Delta manipulator, whose design parameters were given in Table 1 is computed again using the improved algorithm. Employing that decreases the computation time by 41% which shows a good increase in computation speed and efficiency

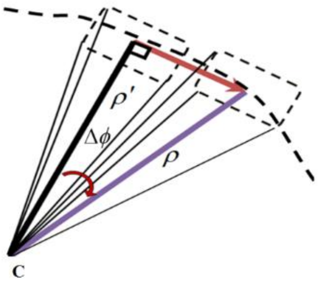


Figure 7. Prediction of the radius of workspace boundary.

3.3. Workspace Volume:

The workspace of Delta robot is defined by a set of points on the boundary centered about a search origin, C. To calculate the workspace volume, the workspace is divided to a set of pyramid sectors as shown in Fig. 9. Then, summation of the volume of the pyramid sectors is provided the total workspace volume. Employing the above procedure for the design parameters of Table 1 gives the workspace volume is 46.3936 (m3).

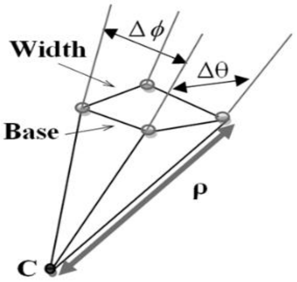


Figure 8. Workspace volume sectors [6].

4. Results and Discussion

Workspace of a Delta robot using a numerical method for a set of design parameters calculated in this article. Then, some improvements for increasing the efficiency of the approach are presented. Using POFILER command in MATLAB, it is possible to investigate the performance of an algorithm. It gives information about the total times of calculation for each function, and number of functions calls during the run time. In this work, to evaluate the performance of the algorithms, the number of functions calls for all of the used functions and their total calculation times compared.

Table 2. Performance of the Modified algorithm

Function	Calls	Total Time	Self-Time
Test-pose	79500	45.458 s	9.615 s
sind	944685	19.078 s	19.078 s
cosd	848714	17.653 s	17.653 s
subplot	181	0.127s	0.076 s
atand	117273	0.567 s	0.567 s
asind	234546	0.262 s	0.262 s
hold	181	0.090 s	0.090 s

Table 3. Performance of the Conti et al. Algorithm

Function	Calls	Total Time	Self-Time
Test-Pose	148239	79.951 s	16.725 s
sind	1652880	33.067 s	33.067
cosd	1488170	30.406 s	30.406 s
subplot	181	0.980 s	0.094
atand	195159	1.022s	1.022s
asind	390318	0.453 s	0.453 s
hold	181	0.090 s	0.090 s

Table 2 and Table 3 represent the functions calls and calculations time for them in the program for $\varepsilon = 0.01, \Delta\theta = \Delta\phi = 2^\circ$. By comparing the results, it is clear that the modified algorithm, calls the Test-Pose function 79500 times and the Conti et al. Method calls 148239 times the functions. Hence it is obvious that modified method is more efficient.

The computation time using MATLAB on a computer with Core i5 (2.4 GHZ) processor, employing unmodified algorithm for $\varepsilon = 0.01, \Delta\theta = \Delta\phi = 1^\circ$ is t = 589.1236 seconds. By applying suggested methodology, the computation time decreases to t =246.647 seconds. So the efficiency of the workspace generation increases by 41%. Finally the workspace volume for the design parameters of Table 1 using numerical approximation is calculated.

5. Conclusions

Workspace determination of a Delta robot is the goal of this work. Since the optimization of link lengths are essential and workspace analysis must be run numerous times for that, the efficiency of the methodology takes more attention. This article suggests an efficient algorithm for Workspace generation of the manipulator.

A numerical search method based on an algorithm proposed somewhere else for a Hexapod structure is initially utilized, and then the method is improved to increase its efficiency. The comparison shows 41% faster speed in computation with respect to the original numerical method. Although, the methodology was applied to a Delta manipulator, that is suitable for workspace problem of parallel manipulators as a general methodology.

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