



# Manipulation Control of a Flexible Space Free Flying Robot Using Fuzzy Tuning Approach

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## ABSTRACT

Cooperative object manipulation control of rigid-flexible multi-body systems in space is studied in this paper. During such tasks, flexible members like solar panels may get vibrated that in turn may lead to some oscillatory disturbing forces on other subsystems, and consequently produces error in the motion of the end-effectors of the cooperative manipulating arms. Therefore, to design and develop capable model-based controllers for such complicated systems deriving a dynamics model is required. However, due to practical limitations and real-time implementation, the system dynamics model should require low computations. So, first to obtain a precise compact dynamics model, the Rigid-Flexible Interactive dynamics Modelling (RFIM) approach is briefly introduced. Using this approach, the system is virtually partitioned into two rigid and flexible portions, and a convenient model for control purposes is developed. Next, Fuzzy Tuning Manipulation Control (FTMC) algorithm is developed, and a Space Free-Flying Robotic (SFFR) system with flexible appendages is considered as a practical case that necessitates delicate force exertion by several end-effectors to move an object along a desired path. The SFFR system contains two cooperative manipulators, appended with two flexible solar panels. The system also includes a third and fourth arm, i.e. a turning antenna and a moving camera. To reveal the merits of the developed model-based controller, the manoeuvre is deliberately planned such that flexible modes of solar panels get stimulated due to arms motion. Obtained results show the effective performance of the proposed approach as will be discussed.

## 1. Introduction

Robotic systems are widely used in unsafe, costly, and repetitive tasks in terrestrial and space applications, [1]. Most available robotic systems are designed such that they can provide essential stiffness for end-effector to reach its desired position without flexible deformations. This stiffness is usually attained by massive parts. Consequently, design and use of weighty rigid members may be deficient in energy consumption and the speed of operation. In particular, for space and on-orbit applications minimum weight design does not allow using stiff and rigid members, [2-3].

In fact, Space Free-Flying Robots (SFFR) have been proposed for on-orbit manoeuvres in which the base body is a satellite with limited mass equipped with continuous thrusters. So, it does respond freely to dynamic reaction forces due to the arms motion. Hence in order to control such a system, it is essential to consider the dynamic coupling between the arms and the base. On the other hand, existence of flexible components on SFFR such as solar panels, necessitates considering their effect.

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The required settling time for vibration of such parts may delay the operation and so conflicts with increasing time efficiency of the system. This conflict of high speed and high accuracy during any operation makes these robots a disputative research problem, [4-5]. Robotic systems with flexible components include continuous dynamic systems that are simplified using a finite number of rigid degrees of freedom and a limited number of modes, which leads to a set of ordinary and partial differential equations that are usually nonlinear and coupled. Precise solution of these systems in most cases is almost impossible, [6]. In studying these systems, if the flexibility effects in mathematic model are ignored, two types of error will be produced. The first one is related to the actuator torques, and the second one corresponds to the position of end-effectors, [7-8]. The position/orientation of end-effectors for precise tasks should not experience any vibration even with small amplitude. Therefore, to achieve high accuracy, more precise mathematical models must be used, [9-11].

On the other hand, control of rigid-flexible multi-body systems is currently an attractive research subject because of its application in flexible field mobile robotic systems and the articulated space structures, [12-14]. This depends on determining the actuator torques that can produce the desired motion of such a complicated multi-body system. In other words, the inverse dynamics is a part of controller design, though control can be directly applied on a physical system without using a numerical model, [15-16]. In fact, operational problems with robotic systems due to structural flexibility leads to subsequent difficulties with their position control, and has been widely studied, [17]. However, force interaction with the environment makes a more challenging problem than the position control of such systems. So, controller design for multi-body systems with flexible members requires development of proper dynamics model of such systems.

To study the dynamics of a rigid-flexible multi-body system, an inertia frame is used as universal reference frame. Moreover, an intermediate reference frame is attached to each flexible or rigid body which is usually called floating frame. The motion relative to this intermediate frame for flexible parts occurs because of the body deformation only. This selection simplifies the calculations of internal forces since the magnitude of the stress and strain do not varies under the rigid body motion. To develop dynamics model of such systems various approaches have been used, including Lagrange method, [9, 18], Hamilton principal, [19], Newton-Euler equations, [20], the virtual work principal, [21], and Kane method, [22]. In fact, such models are also required to be as concise as possible for implementation of model-based control algorithms. In most researches on dynamics analyses, the modelling approach introduces an accumulation in the dynamics of rigid-flexible multi-body systems, [23-24]. So, a variety of methods are used

in these dynamics modelling approaches, while the modelling approach does not affect their non-model based control design.

In this paper, first the system dynamics is virtually partitioned into two rigid and flexible portions, and a convenient model for control purposes of rigid-flexible multi-body systems is developed. Next, to perform a cooperative object manipulation task, the Fuzzy Tuning Manipulation Control (FTMC) is presented. To this end, using a comprehensive simulation routine, a cooperative object manipulation operation is studied for a case study. A Space Free-Flying Robotic (SFFR) system is studied which contains a rigid main body equipped with two manipulating arms and two flexible solar panels. Finally, obtained results of the model verification and the implementations of these controllers on the rigid-flexible multi-body system will be discussed.

## 2. Dynamics Modelling

Rigid-Flexible Interactive Modelling (RFIM) approach has been recently proposed by the authors for dynamics modelling of multi-body systems that decouples motion equations of rigid members from those of flexible members, [25]. Therefore, simpler sets of dynamics equations will be achieved so that the obtained model can be used for model-based controllers. In this section the RFIM approach is briefly described. To this end, using floating frames under the assumption of large displacements and rotations, the situation of each flexible body in the multi-body system is specified by two sets of reference and elastic variables. The reference variables define the situation and the orientation of the considered body while the elastic coordinates describe the body deformations relative to the body reference. To avoid computational difficulties associated with infinite-dimensional spaces, the latter is introduced using classical approximation techniques such as Rayleigh-Ritz method, [15]. So, the kinetic energy of a flexible body is developed and the inertia coupling between the reference motion and the elastic deformation is determined. Subsequently, the motion's equations of the flexible members can be obtained as:

$$\mathbf{M}_r^{(i)} \ddot{\mathbf{q}}^{(i)} + \mathbf{K}^{(i)} \bar{\mathbf{q}}^{(i)} = \mathbf{Q}_e^{(i)} + \mathbf{Q}_v^{(i)}, \quad \{i\} = \{1, 2, \dots, n_b\} \quad (1)$$

where  $n_b$  is the total number of the flexible bodies in the multi-body system. Also,  $\mathbf{Q}_v^{(i)}$  and  $\mathbf{Q}_e^{(i)}$  are correspondingly a quadratic velocity vector which contains all gyroscopic and Coriolis components and the vector of generalized forces associated with the  $\{i\}$ -th body. Moreover,  $\mathbf{M}_r^{(i)}$  and  $\mathbf{K}^{(i)}$  are respectively recognized as the symmetric mass matrix and the symmetric positive definite stiffness matrix of the body  $\{i\}$ . It is recommended that  $\bar{\mathbf{q}}^{(i)}$  is the vector of reference and elastic coordinates of the flexible body. This equation can be written in a partitioned matrix form as:

$$\begin{bmatrix} \mathbf{m}_r^{(i)} & \mathbf{m}_{rf}^{(i)} \\ \mathbf{m}_{rf}^{(i)} & \mathbf{m}_f^{(i)} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_r^{(i)} \\ \ddot{\mathbf{q}}_f^{(i)} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{k}_{rf}^{(i)} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_r^{(i)} \\ \dot{\mathbf{q}}_f^{(i)} \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}_e^{(i)})_r \\ (\mathbf{Q}_e^{(i)})_f \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}_v^{(i)})_r \\ (\mathbf{Q}_v^{(i)})_f \end{bmatrix} \quad (2)$$

where  $r$  and  $f$  respectively refer to rigid and flexible coordinates of the flexible members. The proposed RFIM approach separates dynamics modelling of the flexible members and the rigid elements of a unified total system, [25]. Then, the coupling between these two sets of equations is realized by the kinematics constraints so that the two sets of equations are simultaneously solved. So, if it is supposed that the motion's equations of the rigid mobile subsystem can be modified as, [26]:

$$\mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \mathbf{Q}(\mathbf{q}) + \mathbf{Q}_{\text{flex.}}(\mathbf{q}) \quad (3)$$

where  $\mathbf{q}$  describes the generalized coordinates corresponding to the rigid subsystem,  $\mathbf{H}$ ,  $\mathbf{C}$ ,  $\mathbf{G}$ , and  $\mathbf{Q}$  are the corresponding mass matrix, vector of nonlinear velocity terms, vector of gravity related terms, the generalized forces, respectively, and  $\mathbf{Q}_{\text{flex.}}(\mathbf{q})$  is the generalized forces due to the stimulation of the flexible members which are applied on the rigid subsystem as the modification term or the constraint force. Considering Eq. (2), this term can be achieved as:

$$\mathbf{Q}_{\text{flex.}}(\mathbf{q}) = \sum_{\substack{\{i\}=\{1\} \\ \{i\}=\{n_b\}}} \mathbf{J}_f^{(i)T} (\mathbf{Q}_e^{(i)})_r \quad (4)$$

where  $\mathbf{J}_f^{(i)}$  is the Jacobean matrix of the floating frame of each flexible body related to the inertial frame of the main body. As detailed above, the RFIM approach combines the Lagrange and Newton-Euler methods. To use the RFIM approach, the computation procedure at each time step includes the following calculations. First, the motion's equations of the rigid subsystem or Eq. (3) are solved, and the acceleration, velocity and position terms of the rigid subsystem, i.e.  $\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}$  in the above formulation, are calculated. Then, the rigid components of the acceleration, velocity and position terms of each flexible body, i.e.  $\ddot{\mathbf{q}}_r^{(i)}, \dot{\mathbf{q}}_r^{(i)}, \mathbf{q}_r^{(i)}$  in the above formulation, are computed and inserted into the motion equations of the flexible members as input terms. The relationship between these two sets of variables is established by the kinematics constraints between the origin of the floating frame which is attached to the flexible member and the reference frame of the rigid subsystem. Considering these inputs and by solving the second row of Eq. (2), the flexible components of the acceleration, velocity and position terms of each flexible body, i.e.  $\ddot{\mathbf{q}}_f^{(i)}, \dot{\mathbf{q}}_f^{(i)}, \mathbf{q}_f^{(i)}$  in the above formulation, are calculated. Using these values, and substituting into the first row of Eq. (2), the constraint forces or  $\mathbf{Q}_e^{(i)}$  are computed. Also, these results are applied to the equations of the rigid subsystem as the produced forces from the incitement of the flexible member, i.e.  $\mathbf{Q}_{\text{flex.}}(\mathbf{q})$  using Eq. (4). In the next section,

the Fuzzy Tuning Manipulation Control (FTMC) is stated for a cooperative object manipulation using the RFIM approach.

### 3. Fuzzy Tuning Manipulation Control (FTMC)

To perform a cooperative object manipulation task by rigid-flexible robotic systems, the flexible members must be controlled to minimize the disturbance forces which are produced from the incitement of the fast dynamics of these flexible members. Also, in order to complete the successful operation it is necessary to have a more precise control on the members which are involved in the task. So, this is considered as the primary key to study the Fuzzy Tuning Manipulation Control (FTMC), [27]. Therefore, the flexible member oscillations are controlled using this controller to have minimum disturbances on the mobile base due to those oscillations, which results in a perfect tracking for the system variables during an object manipulation operation. Forasmuch as these flexible members naturally have the passive dynamics, therefore they are not directly controllable. So, controlling the parts of the robotic system which they result in the stimulation of the passive flexible members in such a manner that the object manipulation operations do not interrupt is the finest solution. This concept can be named as the Fuzzy Tuning. In fact, tuning of the controller gain of the robotic system parts while the flexible members install on them and they perform the object manipulation task (or involved parts), is the purpose of the FTMC. This result in the path changes of the object manipulation of the robotic system into a form that the passive flexible members are stimulated at least possible bound. So, the FTMC causes the balance between the error of the involved parts and the stimulated amount of the passive flexible members in the object manipulation operation. Next, the FTMC based on the achieved dynamics model in the previous section is studied. To this end, the motion's equations of the robotic system or Eq. (3) can be written in the task space as:

$$\tilde{\mathbf{H}}^{(i)}(\mathbf{q}^{(i)}) \ddot{\tilde{\mathbf{X}}}^{(i)} + \tilde{\mathbf{C}}^{(i)}(\mathbf{q}^{(i)}, \dot{\mathbf{q}}^{(i)}) = \tilde{\mathbf{Q}}^{(i)} + \tilde{\mathbf{Q}}_{\text{flex.}}^{(i)} \quad (5)$$

where  $(i)$  indicates the  $(i)$ -th manipulator and:

$$\begin{aligned} \tilde{\mathbf{H}}^{(i)} &= \mathbf{J}_c^{(i)T} \mathbf{H}^{(i)} \mathbf{J}_c^{(i)} \\ \tilde{\mathbf{C}}^{(i)} &= \mathbf{J}_c^{(i)T} \mathbf{C}^{(i)} - \dot{\tilde{\mathbf{H}}}^{(i)} \mathbf{J}_c^{(i)} \dot{\mathbf{q}}^{(i)} \\ \tilde{\mathbf{Q}}^{(i)} &= \mathbf{J}_c^{(i)T} \mathbf{Q}^{(i)} \quad , \quad \tilde{\mathbf{Q}}_{\text{flex.}}^{(i)} = \mathbf{J}_c^{(i)T} \mathbf{Q}_{\text{flex.}}^{(i)} \end{aligned} \quad (6)$$

where  $\mathbf{J}_c^{(i)}$  is Jacobian matrix for the  $(i)$ -th manipulator. Also,  $\tilde{\mathbf{X}}^{(i)}$  is the output coordinate. The main principle of the Multiple Impedance Control (MIC), [28], is that the robot base, end-effectors, and the object must be moved accordingly for a cooperative object manipulation. Thus, using this approach, the required forces for the object manipulation to be supplied by actuators are:

$$\tilde{\mathbf{Q}}^{(i)} = \tilde{\mathbf{Q}}_m^{(i)} + \tilde{\mathbf{Q}}_f^{(i)} + \tilde{\mathbf{Q}}_{\text{react}}^{(i)} + \tilde{\mathbf{Q}}_{\text{supp.}}^{(i)} \quad (7)$$

where  $\tilde{\mathbf{Q}}_m^{(i)}$  is the control forces for the end-effector motion,  $\tilde{\mathbf{Q}}_{react}^{(i)}$  is the reaction load on the end-effectors and virtually cancelled the control forces for the object motion or  $\tilde{\mathbf{Q}}_f^{(i)}$  and  $\tilde{\mathbf{Q}}_{supp}^{(i)}$  is the suppression control forces for the flexible members. If the motion's equations of the object are stated as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{F}_w = \mathbf{F}_c + \mathbf{F}_o + \mathbf{G}\mathbf{F}_e \quad (8)$$

where  $\mathbf{G}$  is the grasp matrix,  $\mathbf{F}_w$  is the vector of nonlinear velocity terms,  $\mathbf{F}_c$  is the desired exerted forces from end-effectors,  $\mathbf{F}_o$  describes other external forces that act on the object and  $\mathbf{F}_e$  is the contact force. Choosing the Control Law for the object motion as:

$$\mathbf{v} = \ddot{\mathbf{x}}_d + \mathbf{K}_d(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}) \quad (9)$$

where  $\mathbf{e} = \mathbf{x}_d - \mathbf{x}$  is the tracking error of object variables and  $\mathbf{K}_d, \mathbf{K}_p$  are the gain matrices for the proposed controller. Then, the desired forces to move the object is obtained as:

$$\mathbf{F}_{e_{req}} = \mathbf{G}^\# \{ \mathbf{M}\mathbf{v} + \mathbf{F}_w - (\mathbf{F}_c + \mathbf{F}_o) \} \quad (10)$$

where  $\mathbf{G}^\#$  is the pseudo inverse of  $\mathbf{G}$ . So, the desired exerted forces from end-effectors to move the object is directly achieved from this force as:

$$\tilde{\mathbf{Q}}_f^{(i)} = \mathbf{F}_{e_{req}} \quad (11)$$

Similarly, choosing the same Law for each end-effector as:

$$\tilde{\mathbf{v}}^{(i)} = \ddot{\tilde{\mathbf{x}}}_d^{(i)} + \tilde{\mathbf{K}}_d(\dot{\tilde{\mathbf{x}}}_d^{(i)} - \dot{\tilde{\mathbf{x}}}^{(i)}) + \tilde{\mathbf{K}}_p(\tilde{\mathbf{x}}_d^{(i)} - \tilde{\mathbf{x}}^{(i)}) \quad (12)$$

where  $\tilde{\mathbf{e}}^{(i)} = \tilde{\mathbf{x}}_d^{(i)} - \tilde{\mathbf{x}}^{(i)}$  is the system tracking error, and  $\tilde{\mathbf{K}}_d, \tilde{\mathbf{K}}_p$  are the gain matrices. Thus, the required force to move end-effectors is expressed as:

$$\tilde{\mathbf{Q}}_m^{(i)} = \tilde{\mathbf{H}}^{(i)}(\mathbf{q}^{(i)})\tilde{\mathbf{v}}^{(i)} + \tilde{\mathbf{C}}^{(i)}(\mathbf{q}^{(i)}, \dot{\mathbf{q}}^{(i)}) \quad (13)$$

It should be noted that  $\tilde{\mathbf{x}}_d^{(i)}$  and  $\tilde{\mathbf{x}}^{(i)}$  are attained based on the design trajectory for the object motion and the grasp condition. Moreover, the linear dynamics of the error is obtained by substituting Eqs. (13) and (11) into Eq.(7), and then the result into Eq. (5) yields:

$$\tilde{\mathbf{H}}^{(i)}(\mathbf{q}^{(i)}) \left\{ \tilde{\mathbf{x}}^{(i)} - \left[ \tilde{\mathbf{x}}_d^{(i)} + \tilde{\mathbf{K}}_d(\dot{\tilde{\mathbf{x}}}_d^{(i)} - \dot{\tilde{\mathbf{x}}}^{(i)}) + \tilde{\mathbf{K}}_p(\tilde{\mathbf{x}}_d^{(i)} - \tilde{\mathbf{x}}^{(i)}) \right] \right\} = \mathbf{0} \quad (14)$$

$$\mathbf{M} \left\{ \ddot{\mathbf{x}} - \left[ \ddot{\mathbf{x}}_d + \mathbf{K}_d(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}) \right] \right\} = \mathbf{0}$$

Considering the fact that  $\tilde{\mathbf{H}}^{(i)}$  and  $\mathbf{M}$  are the positive definite matrices, this result in:

$$\begin{aligned} \ddot{\tilde{\mathbf{e}}}^{(i)} + \tilde{\mathbf{K}}_d \dot{\tilde{\mathbf{e}}}^{(i)} + \tilde{\mathbf{K}}_p \tilde{\mathbf{e}}^{(i)} &= \mathbf{0} \\ \ddot{\mathbf{e}} + \mathbf{K}_d \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} &= \mathbf{0} \end{aligned} \quad (15)$$

In the FTMC, the gains of the robot and the object control law or  $\tilde{\mathbf{K}}_d, \tilde{\mathbf{K}}_p$  and  $\mathbf{K}_d, \mathbf{K}_p$  are tuned by the Intelligent Fuzzy Tuning Controller. The Fuzzy Tuning Controller operates based on the two data for the designated path in the object manipulation task. The first is the roots of the characteristic equations of the closed-loop error dynamics or Eq. (15), and the second is the magnitude of the stimulated error of the passive flexible members in order of the position and velocity. If it is supposed that these passive flexible members are installed on the mobile base of the robotic system, so the controller gains of the robot or  $\tilde{\mathbf{K}}_d, \tilde{\mathbf{K}}_p$  were concentrated. Then, the controller gains of the object are normally designed. Therefore, the fuzzy rules are applied by considering the roots of the characteristic equations of the error dynamics of the robot base and the stimulated magnitude of the passive flexible members or  $\tilde{\mathbf{e}}_f^{(i)} = \tilde{\mathbf{q}}_f^{(i)} - \tilde{\mathbf{q}}_f^{(i)}$ . As an instance:

"If  $\dot{\tilde{\mathbf{e}}}_f^{(i)}$  is Very Large (VL) and  $\tilde{\mathbf{e}}^{(i)}$  is Very Small (VS) then  $\tilde{\mathbf{K}}_d$  is Normal (N)."

or another one as:

"If  $\dot{\tilde{\mathbf{e}}}_f^{(i)}$  is Small (S) and  $\tilde{\mathbf{e}}^{(i)}$  is Large (L) then  $\tilde{\mathbf{K}}_d$  is Normal (N)."

and at last, the inference table of these rules is stated in Table (1) in which VS, S, N, L and, VL are respectively pointed to Very Small, Small, Normal, Large and Very Large amount of each parameters. Also, the inference control surface of these rules is shown in Figure (1) for the assumed Gaussian function for each input and output signals. The same procedure is used to determine the amount of  $\tilde{\mathbf{K}}_p$  via the Fuzzy Tuning Controller. It should be noted that the scaled inference fuzzy system is applied in all of the simulations, [29-31]. A block diagram of the implementation procedure of this control algorithm for the cooperative object manipulation is shown in Figure (2). Next, this object manipulation control algorithm is applied on the robotic system and study it in more details.

Table 1: Rules of Fuzzy inference tuning controller for  $\tilde{\mathbf{K}}_d, \tilde{\mathbf{K}}_p$

$\dot{\tilde{\mathbf{e}}}_f^{(i)} \backslash \tilde{\mathbf{e}}^{(i)}$	VS	S	N	L	VL
VS	VS	VS	S	S	N
S	VS	S	S	N	L
N	S	S	N	L	L
L	S	N	L	L	VL
VL	N	L	L	VL	VL

#### 4. Case Study

Space Free-Flying Robots (SFFRs) are space robotic systems equipped with a single or multiple manipulators and thruster jets on the base satellite. These thrusters were originally proposed as on-off cold jets, while in the recent decades have been introduced as hot continuous ones. Distinct from fixed-based manipulators, the base of a SFFR responds to dynamic reaction forces due to the arms motion. Unlike long reach space manipulators, SFFR are suggested to be comparable to the size of an astronaut, and are usually investigated under the assumption of rigid elements. Here, the SFFR system contains a rigid main body equipped with two manipulating arms and two flexible solar panels. The system also includes a rotating antenna and camera as its third and fourth arm. As mentioned before, the RFIM approach may be interpreted as a method that combines the Lagrange and Newton-Euler methods. So, the coupling between the rigid subsystem and the flexible one is restricted to the constraint forces which are computed at each time step, and applied to the rigid part for computations of the next time step. This compact model of the rigid-flexible multi-body systems can be used to study the effects of the dynamics terms in the incitement of flexible members. Also, in applying a model-based control algorithm such as FTMC for a cooperative object manipulation control, this compact model is highly useful. These points are discussed in the following sections.

##### 5.1 Dynamics Modelling of SFFR

The closed-form dynamics equations of a SFFR with just rigid components were obtained and verified in [32]. Considering the flexible appendages with the specifications described in [33], the dynamics model of the rigid SFFR of Figure (3) is modified and expressed as, [25]:

$$\mathbf{H}(\boldsymbol{\beta}_0, \boldsymbol{\theta}) \ddot{\mathbf{q}} + \mathbf{C}_1(\boldsymbol{\beta}_0, \dot{\boldsymbol{\beta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \dot{\mathbf{q}} + \mathbf{C}_2(\boldsymbol{\beta}_0, \dot{\boldsymbol{\beta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{Q}(\boldsymbol{\beta}_0, \boldsymbol{\theta}) + \mathbf{Q}_{\text{flex}}(\boldsymbol{\beta}_0) \quad (16)$$

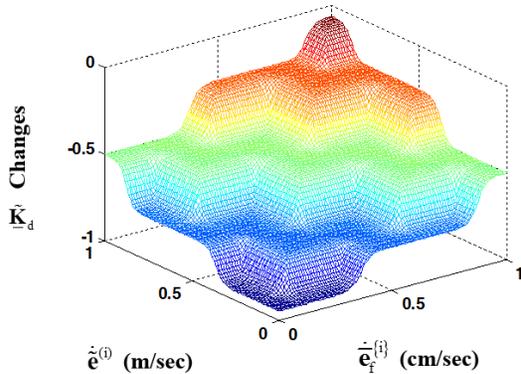


Figure 1: The inference control surface of the Fuzzy Tuning controller

where  $\mathbf{Q}_{\text{flex}}(\boldsymbol{\beta}_0)$  is the generalized resultant forces/torques applied on the main body of the SFFR due to vibrating motion of the flexible solar panels, and others are fully described in [32]. Next, using the RFIM approach in prior section for modelling the flexible members, the solar panels of the SFFR can be considered as a beam in the planar analysis. The main problem in this modelling is to select the appropriate displacement field or the shape functions for flexible bodies. Therefore, assuming a fixed end beam for the panel, a displacement field is chosen as below:

$$\mathbf{S}^{(i)} = \begin{bmatrix} \xi & 0 & 0 \\ 0 & 3\xi^2 - 2\xi^3 & L_b(\xi^3 - \xi^2) \end{bmatrix} \quad (17)$$

where  $\xi$  is a dimensionless variable defined as  $\xi = x/L_b$  and  $L_b$  is the length of beam. Then,  $\mathbf{L}^{(i)}$  matrix for absolute velocity of the flexible beam is obtained, that leads to computation of the mass matrix for flexible members, i.e. the left and right solar panels, as:

$$\mathbf{M}^{(i)} = \begin{bmatrix} 0 & \frac{m^{(i)}}{12}(6L_b + \bar{q}_4^{(i)}S\delta_0^{(i)} + (6\bar{q}_5^{(i)} - L_b\bar{q}_6^{(i)})C\delta_0^{(i)}) & \frac{m^{(i)}}{2}C\delta_0^{(i)} & \frac{m^{(i)}}{2}S\delta_0^{(i)} & \frac{m^{(i)}L_b}{12}S\delta_0^{(i)} \\ m^{(i)} & \frac{m^{(i)}}{12}(6L_b + \bar{q}_4^{(i)}C\delta_0^{(i)} + (6\bar{q}_5^{(i)} - L_b\bar{q}_6^{(i)})S\delta_0^{(i)}) & \frac{m^{(i)}}{2}S\delta_0^{(i)} & \frac{m^{(i)}}{2}C\delta_0^{(i)} & -\frac{m^{(i)}L_b}{12}C\delta_0^{(i)} \\ m^{(i)}(\frac{L_b^2}{3} + \frac{2}{3}L_b\bar{q}_4^{(i)} + \frac{1}{3}\bar{q}_5^{(i)^2} + \frac{13}{35}\bar{q}_6^{(i)^2}) & \frac{m^{(i)}}{20}(-7\bar{q}_5^{(i)} + L_b\bar{q}_6^{(i)}) & \frac{m^{(i)}}{20}(-7L_b + 7\bar{q}_5^{(i)}) & -\frac{m^{(i)}}{20}(L_b^2 + L_b\bar{q}_4^{(i)}) & 0 \\ -\frac{13}{105}\bar{q}_5^{(i)^2} + \frac{11}{105}L_b\bar{q}_6^{(i)} & 0 & 0 & 0 & 0 \\ \text{Symmetric} & & & & \frac{13}{35}m^{(i)} & \frac{11}{210}m^{(i)}L_b \\ & & & & & \frac{m^{(i)}L_b^2}{105} \end{bmatrix} \quad (18)$$

where  $\bar{q}_4^{(i)}, \bar{q}_5^{(i)}, \bar{q}_6^{(i)}$  are the vector components of elastic coordinates of the flexible body  $\{i\}$ ,  $C\delta_0^{(i)}$  stands for  $\cos\delta_0^{(i)}$ , and correspondingly  $S\delta_0^{(i)}$  for  $\sin\delta_0^{(i)}$ . Based on the system depicted in Figure (3), it yields:

$$\begin{aligned} \{i\} = \{1\} \text{ or } \{L\} \text{ (Left Solar Panel)} &\rightarrow \delta_0^{(L)} = \beta_0 + \pi/2 \\ \{i\} = \{2\} \text{ or } \{R\} \text{ (Right Solar Panel)} &\rightarrow \delta_0^{(R)} = \beta_0 - \pi/2 \end{aligned} \quad (19)$$

where  $\beta_0$  is the yaw angle of the base. Then, if we ignore the shear deformation, based on Euler-Bernoulli beam theory, the stiffness matrix is obtained as:

$$\mathbf{k}_{\text{ff}}^{(i)} = \begin{bmatrix} \frac{Ea}{L_b} & 0 & 0 \\ 0 & \frac{12EI}{L_b^3} & -\frac{6EI}{L_b^2} \\ 0 & -\frac{6EI}{L_b^2} & \frac{4EI}{L_b} \end{bmatrix} \quad (20)$$

where  $E$  is the coefficient of elasticity,  $I$  is the second moment of area for the flexible solar panels, and "a" is the cross sectional of the beam. Also, the vector of quadratic velocity terms which results from

differentiating the kinetic energy with respect to the body coordinates and time is obtained as:

$$\mathbf{Q}_e^{(i)} = \begin{cases} \frac{m^{(i)}\delta_0^2}{12} [6(L_6 + \bar{q}_i^{(i)})C\delta_0^{(i)} - 6\bar{q}_i^{(i)}L_6\bar{q}_i^{(i)}]S\delta_0^{(i)} + \frac{m^{(i)}\delta_0}{6} [6\dot{\bar{q}}_i^{(i)}S\delta_0^{(i)} + (6\bar{q}_i^{(i)}L_6\dot{\bar{q}}_i^{(i)})C\delta_0^{(i)}] \\ \frac{m^{(i)}\delta_0^2}{12} [6(L_6 + \bar{q}_i^{(i)})S\delta_0^{(i)} + (6\bar{q}_i^{(i)}L_6\bar{q}_i^{(i)})C\delta_0^{(i)}] + \frac{m^{(i)}\delta_0}{6} [-6\dot{\bar{q}}_i^{(i)}C\delta_0^{(i)} + (6\bar{q}_i^{(i)}L_6\dot{\bar{q}}_i^{(i)})S\delta_0^{(i)}] \\ -2m^{(i)}\delta_0 [\frac{1}{3}\dot{\bar{q}}_i^{(i)} + \frac{1}{3}\bar{q}_i^{(i)}\dot{\bar{q}}_i^{(i)} + \frac{13}{35}\bar{q}_i^{(i)}\dot{\bar{q}}_i^{(i)} - \frac{11}{210}L_6\bar{q}_i^{(i)}\dot{\bar{q}}_i^{(i)} - \frac{11}{210}L_6\dot{\bar{q}}_i^{(i)}\bar{q}_i^{(i)} + \frac{L_6^2}{105}\dot{\bar{q}}_i^{(i)}\bar{q}_i^{(i)}] \\ m^{(i)}\delta_0^2 [\frac{1}{3}\dot{\bar{q}}_i^{(i)} + \frac{1}{3}\bar{q}_i^{(i)}\dot{\bar{q}}_i^{(i)}] + 2m^{(i)}\delta_0 [\frac{7}{20}\dot{\bar{q}}_i^{(i)} - \frac{L_6}{20}\dot{\bar{q}}_i^{(i)}] \\ m^{(i)}\delta_0^2 [\frac{13}{35}\bar{q}_i^{(i)} - \frac{11}{210}L_6\bar{q}_i^{(i)}] + 2m^{(i)}\delta_0 [\frac{7}{20}\dot{\bar{q}}_i^{(i)}] \\ m^{(i)}\delta_0^2 [\frac{L_6^2}{105}\bar{q}_i^{(i)} - \frac{11}{210}L_6\bar{q}_i^{(i)}] + 2m^{(i)}\delta_0 [\frac{1}{20}\dot{\bar{q}}_i^{(i)}] \end{cases} \quad (21)$$

As explained before, using the RFIM, the motion's equations for the  $\{i\}$ -th flexible body is separated from those of the main rigid body subsystem of the SFFR depicted in Figure (3). Also, the relationship between the generalized variables of the SFFR and the origin of the reference coordinates for the  $\{i\}$ -th flexible body, or the two points on the rigid body can be written as:

$$\bar{\mathbf{R}}_{O^{(i)}} = \bar{\mathbf{R}}_{C_0} + \bar{\mathbf{R}}_{O^{(i)}/C_0} \quad (22)$$

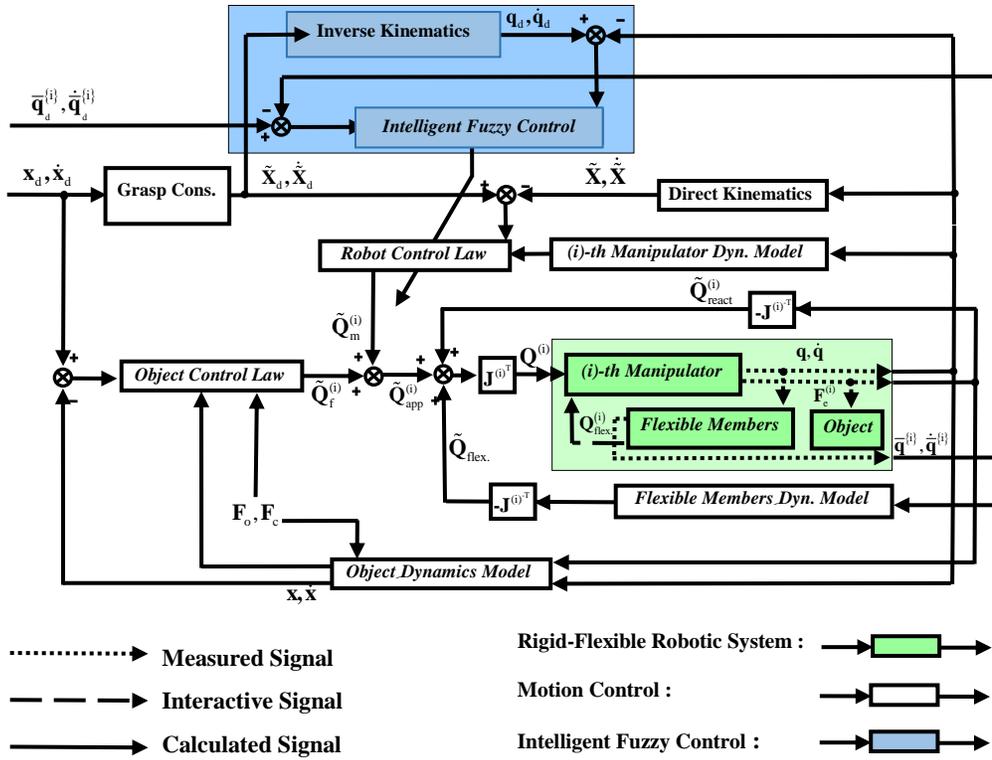


Figure 2: Block diagram of RFIM and FTMC algorithm for cooperative object manipulation

Differentiating this equation, the corresponding velocity and acceleration is obtained. These relationships must be considered in solving the rigid model dynamics together with the flexible subsystem. Therefore, in the first stage the motion's equations of the rigid SFFR of Eq. (16) is solved to compute  $\ddot{\mathbf{q}}, \dot{\mathbf{q}}$  and  $\mathbf{q}$ . Next, based on Eq. (22) and its derivatives which establish the relationship between the generalized variables of the rigid SFFR and the generalized variables of the flexible members,  $\ddot{\bar{\mathbf{q}}}_f^{(i)}, \dot{\bar{\mathbf{q}}}_f^{(i)}$  and  $\bar{\mathbf{q}}_f^{(i)}$  are computed. Then, these

are inserted into the equations of the flexible members as the input terms to calculate  $\ddot{\bar{\mathbf{q}}}_f^{(i)}, \dot{\bar{\mathbf{q}}}_f^{(i)}$  and  $\bar{\mathbf{q}}_f^{(i)}$ . Finally, the inverse dynamics of the  $\{i\}$ -th flexible body or Eq. (1) must be solved to compute  $\mathbf{Q}_e^{(i)}$ . Considering the last three equations of this equation, the deformations of each flexible solar panel or  $\bar{\mathbf{q}}_f^{(i)}$  are calculated. Next, the forces of fixed end beam or the constraint forces  $\mathbf{Q}_c^{(i)}$  are computed as:

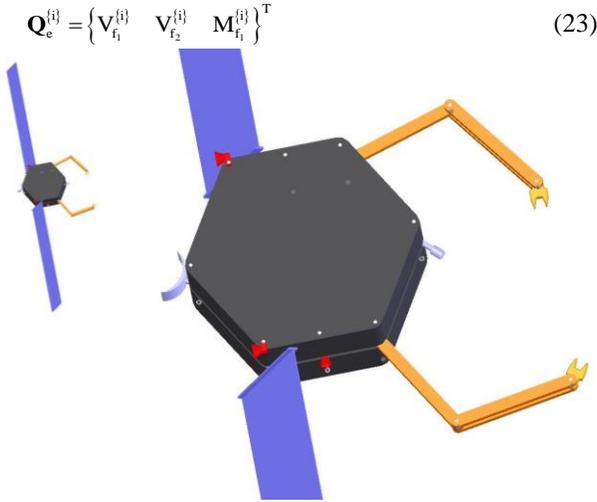


Figure 3: The considered space robotic system which includes two flexible solar panels

So, these forces are calculated using the first three equations of the inverse dynamics of the flexible body, which are applied on the moving base of the SFFR as shown in Figure (3):

$$\begin{aligned} Q_{flex.}^x &= V_{f_2}^{(L)} - V_{f_2}^{(R)} \\ Q_{flex.}^y &= V_{f_1}^{(R)} - V_{f_1}^{(L)} \\ M_{flex.} &= (M_{f_1}^{(R)} + M_{f_1}^{(L)}) - (V_{f_2}^{(L)} + V_{f_2}^{(R)}) \cdot (L \sin(\pi/3)) \\ &\quad + (V_{f_1}^{(L)} - V_{f_1}^{(R)}) \cdot (L \cos(\pi/3)) \end{aligned} \quad (24)$$

where  $|\tilde{\mathbf{R}}_{O^i|C_0}| = L$  and in this case study  $\mathbf{Q}_{flex.}(\beta_0)$  can be written as:

$$\mathbf{Q}_{flex.}(\beta_0) = \{Q_{flex.}^x \quad Q_{flex.}^y \quad M_{flex.}\}^T \quad (25)$$

which completes the procedure of dynamics modelling of the complicated SFFR system, using RFIM approach.

## 5.2 Obtained Results

Considering the geometric and mass parameters for the main rigid system which its specifications parameters are addressed in Table 2, two solar panels as described in Table 3, are appended to the base. Also, an operation along a Circular Path with a radius of 15 m is considered, and different scenarios to perform the operation are studied. Considering the mobile base and a desired path length, several solutions exist. By using the inference rules of the scaled Fuzzy Tuning controller of Table (1), the  $\tilde{\mathbf{K}}_d$  and  $\tilde{\mathbf{K}}_p$  gains of the FTMC are varied (Figure 4). These changes minify the roots of the characteristic equations of the closed-loop error dynamics. As shown in Figures (5-8) for the work space and object variables and their rates (not for the involved variable, i.e., the variables of the robot base), this control algorithm can successfully perform the object manipulation task. Simulation results show that the object manipulation

operation causes vibration of the flexible solar panels while the FTMC controlled them, where the time history of these deflections is shown in Figure 9 for a few time steps.

Table 2: The specifications parameters of the flexible members, [33]

$m = 9$ (Kg)	$\rho = 1.4$ (g/cm <sup>3</sup> )
$L_b = 4$ (m)	$E = 60$ (GPa)
$a = 16$ (cm <sup>2</sup> )	$EI = 20$ (N.m <sup>2</sup> )

Table 3: The geometric and mass parameters of the assumed SFFR system

i,j		$m_{ij}$ (kg)	$l$ (m)	$I_{z_j}$ (kg.m <sup>2</sup> )
i=1	i=2			
	j			
0		50	1	10
1		4	1	0.5
2		3	1	0.5

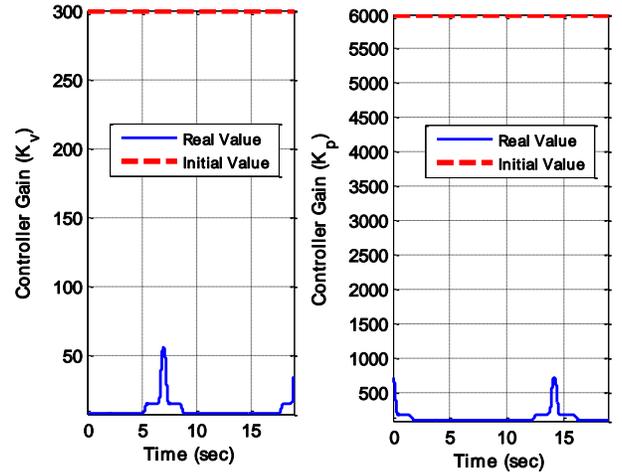


Figure 4: Changing in the controller gains of the robot base using the FTMC

In addition, an animated view of the system performing the object manipulation task along the designed circular path is shown in Figure (10). It should be noted that the error of the involved variables and its rates are restricted and consequently the FTMC results in the perfect object manipulation operation (Figures (11-12)). Also, as shown in Figure (13) for the end point of the solar panels, the passive dampers can be placed on the flexible solar panels to decrease the vibration amplitude of them. Also, it should be noted that it is supposed that the flexible solar panels possess an initial velocity as shown in Figure 9.

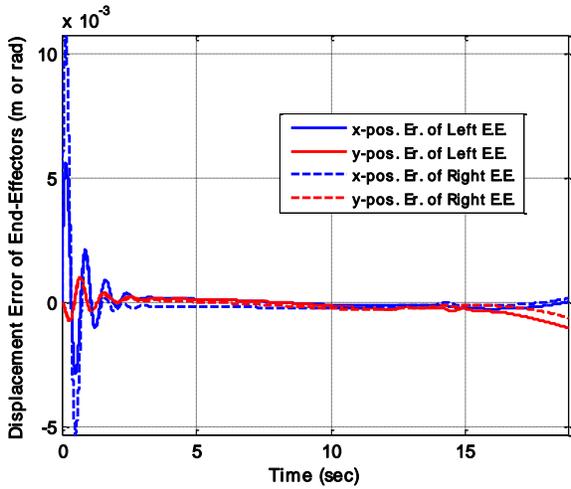


Figure 5: Error of the work space variables during the task

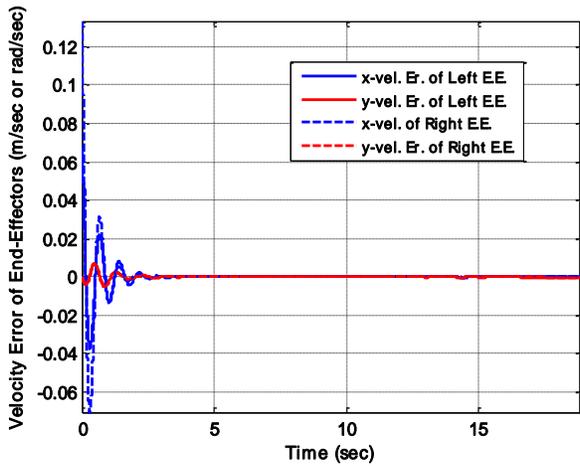


Figure 6: Error of the work space variables rates during the task

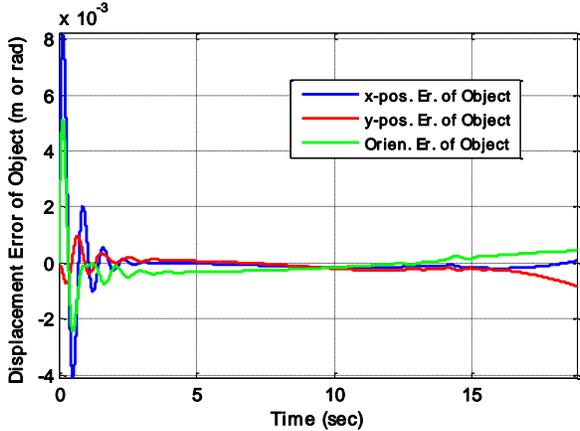


Figure 7: Error of the object variables during the task

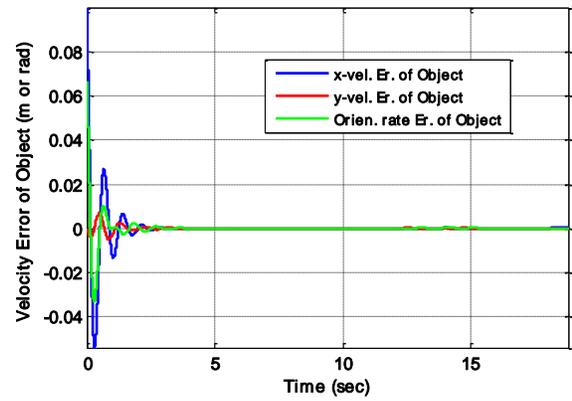


Figure 8: Error of the object variables rates during the task

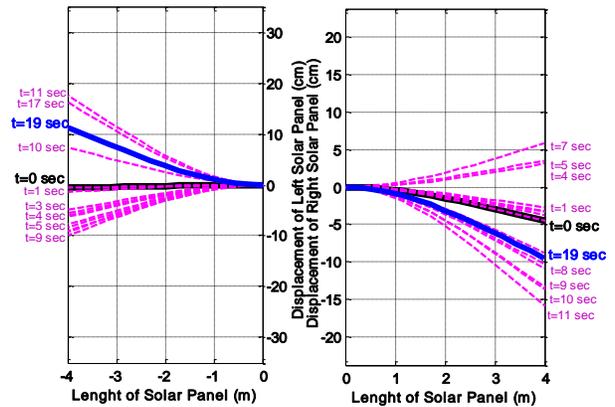


Figure 9: Deflection time history of the passive flexible solar panels

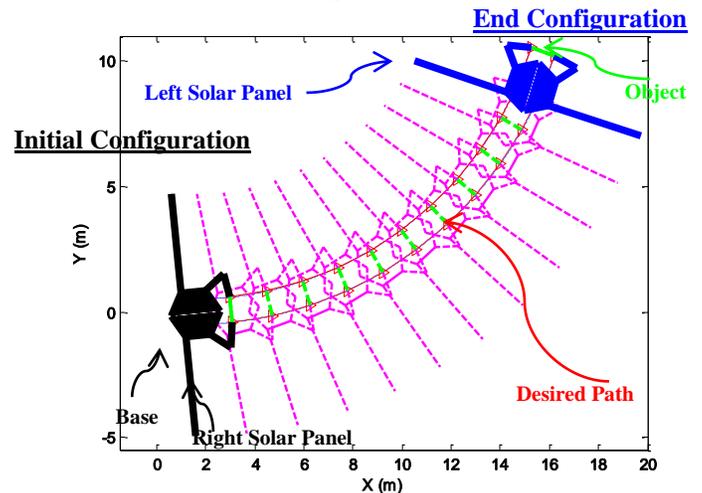


Figure 10: An animated view of the system during the object manipulation task

## 6. Conclusions

In this article, Fuzzy Tuning Manipulation Control (FTMC) of cooperative object manipulation by rigid-flexible multi-body systems was studied. To this end, first the Rigid-Flexible Interactive Modelling (RFIM) approach for dynamics modelling of such systems was reviewed. Using the RFIM approach, the motion equations of rigid and flexible members are separately

developed, assembled and solved simultaneously at each time step by considering the mutual interaction and constraint forces.

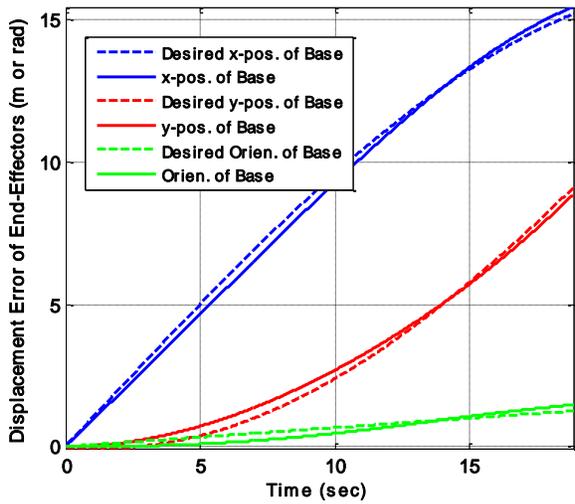


Figure 11. Desired and real amount of the involved variables during the task

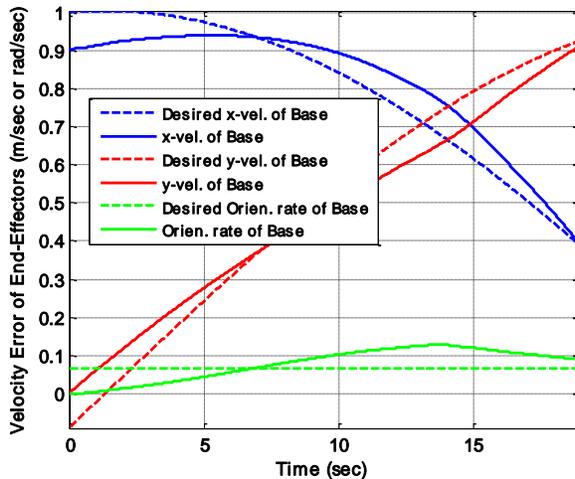


Figure 12. Desired and real amount of the involved variables rates during the task

Therefore, the compact dynamics model which is extracted using the RFIM approach is extremely useful for simulation studies in controller design, and also practical implementations. It was emphasized that control of such systems is highly complicated due to severe under-actuated condition caused by flexible elements, and an inherent uneven nonlinear dynamics. Therefore, the FTMC algorithm was presented next, where its stability and the error convergence were proven. Finally, to reveal the merits of this new control algorithm, its application on a Space Free-Flying Robotic (SFFR) system was studied. Obtained results of the implementations of this controller on the rigid-flexible multi-body system were discussed. It was shown that vibration of the passive flexible solar panels results in generalized forces which can produce undesirable errors of the end-effectors. These effects were significantly

eliminated by the FTMC algorithm, and the manipulation task was successfully performed.

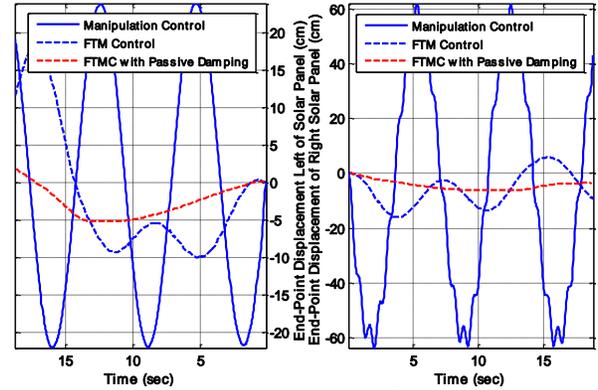


Figure 13. End point displacement of flexible solar panels

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