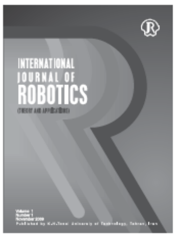

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Modifying a Conventional Grasping Control Approach for Undesired Slippage Control in Cooperating Manipulator Systems

Mehdi Keshmiri[a], Shahram Hadian Jazi[b] and Farid Sheikholeslam[c]

a- Department of Mechanical Eng., Isfahan University of Technology, Isfahan, e-mail: mehdik@cc.iut.ac.ir

b- Faculty of Engineering, Shahrekord University, Shahrekord, e-mail: hadian@eng.sku.ac.ir

c- Department of Electrical and Computer Eng., Isfahan University of Technology, Isfahan, e-mail: sheikh@cc.iut.ac.ir

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ABSTRACT

There have been many researches on object grasping in cooperating systems assuming no object slippage and stable grasp and the control system is designed to keep the contact force inside the friction cone to prevent the slippage. However undesired slippage can occur due to environmental conditions and many other reasons. In this research, dynamic analysis and control synthesis of a cooperating system, considering slipping conditions are performed. Equality and inequality equations of the frictional contact conditions are replaced by a single second order differential equation with switching coefficients in order to facilitate the dynamical modeling and control synthesis. Using this new modeling of friction, a conventional approach in grasping control is modified and presented to control any undesired slippage of the end-effectors on the object.

1- Introduction

Grasping is an important issue in cooperating systems such as multi-fingered hands and multiple robots. Numerous reports can be found on grasp planning. Researches on grasp planning focus on two category problems: grasp analysis and grasp synthesis. In grasp analysis, most of the researchers have focused on finding appropriate conditions for force-closure grasps. Early, Reulaux introduced the notion of force-closure and form-closure [1]. Using screw theory, Salisbury and Roth developed several different types of finger contacts and showed which finger configurations allow complete immobilization of the gripped object relative to the fingers as well as manipulation of the object while maintaining the grasp [2]. With the linearization of the friction cone, Liu developed a ray-shooting based algorithm using the duality of polytopes [3]. Zheng and Qian enhanced the ray-shooting approach proposed by Liu to complete the exactness, increase the efficiency, and extend the scope [4]. With this, the general problem of determining if a grasp is force closure is considered to be completely solved.

Having sufficient conditions for force closure, grasp synthesis deals with optimal grasping. This synthesis consists of: 1) determination of the optimality criteria and, 2) derivation of methods and algorithms for computing contact locations with respect to the optimality criteria and accessibility constraints. Liu et al. introduced several candidate grasp quality functions and formulated the grasp synthesis problem as a max-transfer, max-normal-grasping-force, and a min-analytical-center problem [5]. Based on the geometric condition of the closure property, Zhu and Ding presented a numerical test to quantify how far a grasp is from losing form/force closure. They also developed an iterative algorithm for computing optimal force-closure grasps [6]. Morales et al. addressed the problem of designing a practical system able to grasp real objects with a three-fingered robot hand. They presented a general approach for synthesizing two and three-finger grasps on planar unknown objects using visual perception [7]. Using linear system theory and the singular value of the output controllability matrix, Yamashima and Yamawaki defined a task-oriented accuracy measure for a cooperative manipulation system. They assumed no slippage condition between finger tip and the grasped object [8]. These researches consider no slippage in grasping, and control system tries to keep the contact forces inside the friction cone.

Zheng et al. addressed dynamic and control analysis of a three-fingered hand manipulating and regrasping an object in 3D space. They allowed one of the fingers to slide on a predefined path on the object surface to change its grasp location [9]. Cole et al. consider control of the sliding motion of the fingertip of a two-fingered hand along the object surface and position and orientation control of the object simultaneously. They assumed that only one specific finger slides on a predefined path on the object surface. Their work is useful for regrasping an object held in a hand [10]. Kao and Cutkosky compared theoretical and experimental sliding motions for a sheet of paper or similar objects on a planar surface, manipulated by a two-fingered hand, using static equilibrium equations [11]. Chong et al. proposed a motion/force planning algorithm for multi-fingered hands manipulating an object of an arbitrary shape using both rolling and sliding contacts. They used a nonlinear optimization approach to calculate the joint velocities and contact forces at each step of time [12].

Although the above studies consider slippage in object regrasping analysis, the slippage should be completely defined in advance. Sliding finger, starting time and duration of slippage, and sliding path are all known in advance. This means that dynamic and control analysis of undesired slippage still remains not properly discussed in the literature. Slippage can occur during the grasping maneuver due to many reasons, including changes in the object geometry, mass, inertia and coefficient of friction. It can happen even when the system involves manipulation of an unknown object. As an example, one can assume the practical case where a cooperative system manipulates a dirty object or an object in dirty environment. In such a case, the coefficient of friction between the end-effectors and the object can be changed.

Authors of this paper discussed the control of undesired slippage in a single arm manipulation in [13] and [14]. Where in [14], the analytical and simulation results are verified with experimental results.

2- Dynamic Analysis

The system under consideration is shown in Fig. 1.

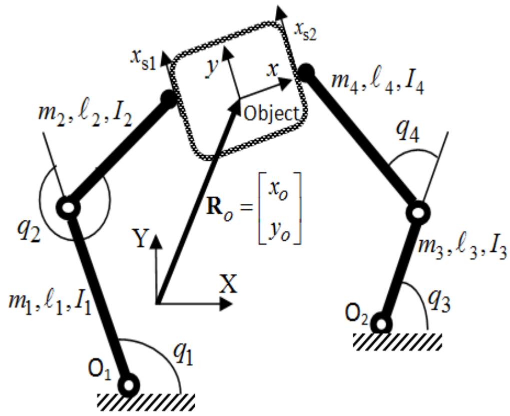

Fig. 1: Schematic of the system.

Each robot arm is a two-link rigid manipulator. The contact between each manipulator and the object is assumed to be point contact which can move along the object surface. Clearly it remains fixed on the end-effector. The whole motion is in the vertical plane and it is assumed there is no uncertainty in the system.

Equations of motion of the system can be presented in the following form

$$\mathbf{M}_i \ddot{\mathbf{q}}_i + \mathbf{h}_i = \boldsymbol{\tau}_i - \mathbf{J}_i^T \mathbf{R} \mathbf{F}_i \quad (i=1,2) \quad , \tag{1}$$

$$\mathbf{M}_o \ddot{\mathbf{q}}_o + \mathbf{h}_o = \mathbf{G}\mathbf{F} \quad , \tag{2}$$

$$\mathrm{H}_i\left(\mathbf{F}_i, \ddot{x}_{si}\right) = 0 \quad (i=1,2) \quad , \tag{3}$$

$$\mathbf{r}_{ei} = \mathbf{R}_o + \mathbf{R}\,\mathbf{r}_{si} \quad (i=1,2) \quad , \tag{4}$$

where $\mathbf{q}_i$ and $\boldsymbol{\tau}_i$ are the generalized coordinates and driving force/torque, respectively. $\mathbf{M}_i$ is inertia matrix and $\mathbf{h}_i$ is gravitational, centrifugal and coriolis terms of a two link serial manipulator. $\mathbf{J}_i$ is the Jacobean matrix of a two link serial manipulator. $\mathbf{F}_i$ is the contact force vector consisting of friction and normal forces exerted by the end-effector on the object. $\mathbf{q}_o$ is the set of generalized coordinates contributed by the object, and $\mathbf{h}_o$ is the contribution of other external forces as well as centrifugal forces of the object. H_i is a function which models friction on the object surfaces, x_{si} is the local sliding state of ith end-effector on the object, $\mathbf{r}_{ei}$ is the position vector of ith end-effector with respect to inertia frame, $\mathbf{R}_o$ is the position vector of the object center of mass with respect to inertia frame, $\mathbf{r}_{si}$ is the ith contact point position vector with respect to object frame, $\mathbf{R}$ is the rotation matrix of object frame w.r.t. inertia frame, $\mathbf{G}$ is the grasping matrix, and finally

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1^T & \mathbf{F}_2^T \end{bmatrix}^T \quad . \tag{5}$$

3-Contact Force Modeling

Assuming the Standard Coulomb friction model without stiction (Fig. 2) with μ as the coefficient of friction ($\mu_S = \mu_k = \mu$), the friction force, exerted on a body from the contacting surface (Fig. 3), can be written as:

$$\begin{cases} F_t = -\mu F_n \operatorname{sign}(v) & \text{if} \quad v \neq 0 \ , \\ |F_t| \leq \mu F_n & \text{if} \quad v = 0 \ \ and \ \ \dot{v} = 0 \ , \\ F_t = 0 & \text{if} \quad v = 0 \ \ and \ \ \dot{v} \neq 0 \ , \end{cases} \tag{6}$$

where v is the speed of the body relative to the surface and F_t and F_n are the friction and normal forces, respectively. F_n is assumed to be positive value.

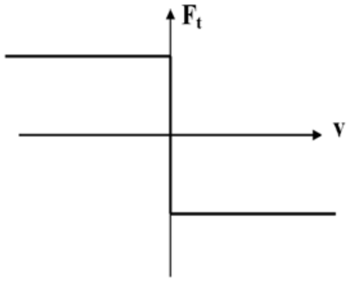

Fig. 2: Coulomb friction model.

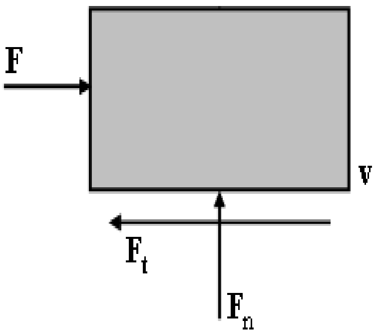

Fig. 3: Free body diagram for a moving object on a surface.

Note that the second equation in

(6) describes three different conditions, starting forward motion, starting backward motion, and stationary condition. We can reformulate the above conditions in a single equation:

$$\alpha_1 \dot{v} + \alpha_2 F_t + \alpha_3 \mu F_n = 0 \quad , \tag{7}$$

where α_i (i=1,2,3) are state dependent coefficients calculated from Table I. When there is more than one choice for α_i (i=1,2,3) we have to choose one and check the consistency of the results from dynamic analysis.

The Now let us consider free body diagram of the object in the cooperating system shown in Fig. 1. This free body diagram is given in Fig. 4

TABLE.1:values for α_i (i=1,2,3) in different conditions

v	$v \neq 0$	$v = 0$				
		$\dot{v}^- \neq 0$		$\dot{v}^- = 0$		
α_i	Motion	Motion reversing	No Motion	No Motion	Start forward	Start backward
α_1	0	0	1	1	0	0
α_2	1	1	0	0	1	1
α_3	sign(v)	0	0	0	1	-1

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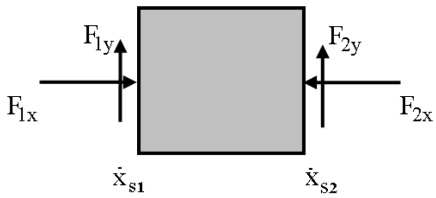

Fig. 4: Free body diagram for the object in Fig. 1.

The contact conditions can be formulated by the following equations:

$$H_1(\mathbf{F}_1, \ddot{x}_{s1}) = \alpha_1 \ddot{x}_{s1} + \alpha_2 \mu_1 F_{1x} + \alpha_3 F_{1y} = 0 \quad , \tag{8}$$

$$H_2(\mathbf{F}_2, \ddot{x}_{s2}) = \beta_1 \ddot{x}_{s2} + \beta_2 \mu_2 F_{2x} + \beta_3 F_{2y} = 0 \quad , \tag{9}$$

where α_i and β_i $(i = 1,2,3)$ are calculated from Table A-I in Appendix A. The results of the above modeling are compared with those of SimMech toolbox of MATLAB which uses differential-algebraic equations. The results are completely the same.

Using the above equations and differentiating (3) with respect to time, dynamics of the whole system can be formulated by the following equations:

$$\mathbf{M}_1 \ddot{\mathbf{q}}_1 + \mathbf{h}_1 = \boldsymbol{\tau}_1 - \mathbf{J}_1^T \mathbf{R} \mathbf{F}_1 \, , \tag{10}$$

$$\mathbf{M}_2 \ddot{\mathbf{q}}_2 + \mathbf{h}_2 = \boldsymbol{\tau}_2 - \mathbf{J}_2^T \mathbf{R} \mathbf{F}_2 \, , \tag{11}$$

$$\mathbf{M}_o \ddot{\mathbf{q}}_o + \mathbf{h}_o = \mathbf{G} \mathbf{F} \, , \tag{12}$$

$$\mathbf{A}_{q_1} \dot{\mathbf{q}}_1 + \mathbf{A}_{o1} \dot{\mathbf{q}}_o + \mathbf{A}_{s1} \dot{x}_{s1} = \mathbf{0} \, , \tag{13}$$

$$\mathbf{A}_{q_2} \dot{\mathbf{q}}_2 + \mathbf{A}_{o2} \dot{\mathbf{q}}_o + \mathbf{A}_{s2} \dot{x}_{s2} = \mathbf{0} \, , \tag{14}$$

$$\alpha_1 \ddot{x}_{s1} + \mathbf{D}_1 \mathbf{F}_1 = 0 \, , \tag{15}$$

$$\beta_1 \ddot{x}_{s1} + \mathbf{D}_2 \mathbf{F}_2 = 0 \, , \tag{16}$$

where

$$\mathbf{D}_1 = [\alpha_2 \mu_1 \quad \alpha_3], \tag{17}$$

$$\mathbf{D}_2 = [\beta_2 \mu_2 \quad \beta_3]. \tag{18}$$

As can be seen, the system is a four-phase dynamical system

- No slippage in the end-effectors,
- Slippage in the left end-effector,
- Slippage in the right end-effector,
- Slippage in the both end-effectors,

and it is over actuated and under actuated in the first and last phase, respectively. In the second phase, the system is a determined system with 4 DOF`s and 4 actuators.

4- Control Synthesis

In most of the studies reported, researchers solve the problem of object manipulation for the case that there is no slippage. In the conventional approach of grasp analysis, the controller is designed such that the manipulators exert the required forces on the object and satisfies the no slipping condition. Modifying this conventional approach, we have extended the previous works for the case that the end-effectors slip on the object.

Consider the equations of motion for the object, (2) and let the desired trajectory of the object be $\mathbf{q}_o^{des}(t)$ and the acceleration of the object is chosen as

$$\ddot{\mathbf{q}}_o = \ddot{\mathbf{q}}_o^{des} + \mathbf{K}_{vo}\dot{\mathbf{e}}_o + \mathbf{K}_{po}\mathbf{e}_o \quad , \tag{19}$$

where $\mathbf{K}_{vo}$ and $\mathbf{K}_{po}$ are constant positive definite matrices and $\mathbf{e}_o = \mathbf{q}_o^{des} - \mathbf{q}_o$. Therefore the following resultant force should be applied on the object by the end-effectors,

$$\mathbf{Q}_{res} = \mathbf{G}\mathbf{F} = \mathbf{M}_o\left(\ddot{\mathbf{q}}_o^{des} + \mathbf{K}_{vo}\dot{\mathbf{e}}_o + \mathbf{K}_{po}\mathbf{e}_o\right) + \mathbf{h}_o \,. \tag{20}$$

The object motion is then governed by

$$\ddot{\mathbf{e}}_o + \mathbf{K}_{vo}\dot{\mathbf{e}}_o + \mathbf{K}_{po}\mathbf{e}_o = \mathbf{0} \quad . \tag{21}$$

This guarantees asymptotic stability of the trajectory tracking for the object.

One has to decompose the resultant force, $\mathbf{Q}_{res}$ into the exerted forces on the object by each manipulator and then control the robots to ensure that the calculated forces for the manipulators are implemented. Due to the redundancy in the driving forces of the object, decomposition of the resultant force leads to the following optimization problem,

$$\begin{aligned} &\text{Minimize} \quad \left\|\mathbf{F}^{des}\right\| \\ &\text{Subject to:} \quad \mathbf{Q}_{res} = \mathbf{G}\mathbf{F}^{des}, \\ &\qquad \mathbf{e}_{Ni}^T\mathbf{F}_i^{des} \ge \eta_i\left\|\mathbf{F}_i^{des}\right\|, \\ &\qquad \mathbf{e}_{Ni}^T\mathbf{F}_i^{des} > 0, \end{aligned} \tag{22}$$

where $\eta_i = 1/\sqrt{1+\mu_i^2}$ and $\mathbf{e}_{Ni}$ $(i=1,2)$ is inward normal direction in i-th contact point.

In (22), we have used $\mathbf{F}^{des}$ instead of $\mathbf{F}$ since the exerted forces by manipulators can differ from this calculated force vector. Contact stability can be deteriorated, once the manipulator cannot exert the desired forces. In this case, the end-effector might slip on the object.

Since the end-effector forces must be controlled in both normal and tangential directions, the usual hybrid position/force control cannot be used. So, we design the controller of the manipulator such that the desired forces are exerted by the end-effector and the following conditions are satisfied in the contact point, i.e.:

$$\ddot{\mathbf{r}}_{ei} = \ddot{\mathbf{r}}_{ci} \quad (i=1,2) \ , \tag{23}$$

where $\mathbf{r}_{ci}$ is i-th contact position vector with respect to inertia frame.

Now we divide the input torques in (1) into two parts, $\boldsymbol{\tau}_i = \boldsymbol{\tau}_{ei} + \boldsymbol{\tau}_{fi}$, where $\boldsymbol{\tau}_{ei}$ and $\boldsymbol{\tau}_{fi}$ are responsible for satisfying the condition presented by (23), which is referred here as no slippage condition, and exerting the calculated force, $\mathbf{F}_i^{des}$ on the object. One can compute $\boldsymbol{\tau}_{fi}$ from static equilibrium condition

$$\boldsymbol{\tau}_{fi} = \mathbf{J}_i^T\mathbf{R}\mathbf{F}_i^{des} \ , \tag{24}$$

and $\boldsymbol{\tau}_{ei}$ from free motion of manipulator ($\mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{h}_i = \boldsymbol{\tau}_{ei}$). In this research, a feedback linearization method is used to control the free motion of the manipulator.

It must be noted that using this approach cannot always generate any arbitrary pair of $\mathbf{F}_i$ and $\ddot{\mathbf{r}}_{ei}$ [15]. However, since $\ddot{\mathbf{r}}_{ei}$ is somehow the result of $\mathbf{F}_i$, the above approach can result in the desired objectives. The fundamental structure of this controller is shown in Fig. 5.

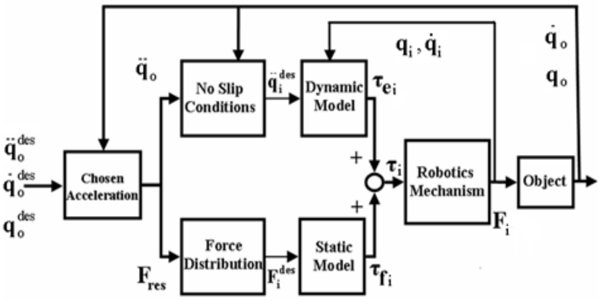

Fig. 5: Structure of the control system.

Our strategy to control the slippage of the end-effectors on the object is to set the velocity of the i-th contact point as the desired velocity for the i-th end-effector and the original contact point position for its desired position, i.e.:

$$\begin{aligned} x_{s1} &= x_{s2} = 0, \\ \dot{x}_{s1} &= \dot{x}_{s2} = 0. \end{aligned} \tag{25}$$

It means that each instant, we try to stop slipping and return the end-effectors to their original positions on the object. Therefore in the motion control part of each manipulator, the desired velocity of the end-effector is the velocity of current contact point on the object while its desired position is the position of the initial contact point on the object. In fact this is the main modification with respect to the conventional approach. Assuming no slippage in the conventional approach, one uses both position and velocity of the current contact point on the object in the motion control of the manipulators.

TABLE.2:Numerical value for parameters

m_j	ℓ_j	m_o	I_o	L_o	$\bar{\mu}_1$	$\bar{\mu}_2$
1(kg)	1(m)	2.5(kg)	0.01042(kgm^2)	0.1(m)	0.25	0.25

where j=1,..,4 and L_o is the vertical distance between the center of mass and edge of the object. The object is assumed to track the following desired trajectory:

$$\ddot{y}_o^{des} = \begin{cases} 0.0256 & 0 < t < 1 \\ 0 & 1 \le t < 6 \\ -0.0256 & 6 \le t < 7 \end{cases}, \quad (26)$$

$$y_o^{des}(0) = 0.366 \;,\; \dot{y}_o^{des}(0) = 0 \;,$$

$$x_o^{des}(t) = 1.466 \;,\; \theta_o^{des}(t) = 0 .$$

In order to simulate the slipping phenomenon, we assume that during the motion, the coefficients of friction between the end-effectors and the object change from its nominal value: given in Table II,

$$\mu_1 = \bar{\mu}_1 \;,\; \mu_2 = \bar{\mu}_2 \quad \text{if} \;\; 0 \le t \le 0.5 \; and \; t > 2 \;,$$
$$\mu_1 = 0.22 \;,\; \mu_2 = 0.23 \;\; \text{if} \;\; 0.5 < t \le 2 \; . \quad (27)$$

Note that the control law is calculated using nominal values.

Performance of the control approach in trajectory tracking, slippage control is shown in Fig. 6 to Fig. 10. The manipulators' torques are shown in Fig. 11. Fig. 12 shows the end-effectors slippage velocity using conventional control method without modification presented in (25). Comparing it with Fig. 10, it can be seen that the system has diverged. It shows that the conventional control approach (mentioned in [15]) cannot control the slippage velocity as soon as slippage happens. Robustness of the controller is also studied numerically with reducing the mass parameters by 20% in the controller. The results are shown in Fig. 13 to Fig. 15.

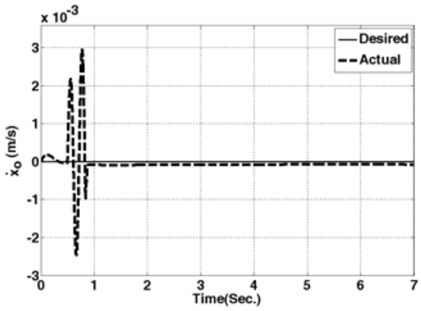

Fig. 6: Horizontal velocity tracking of the object.

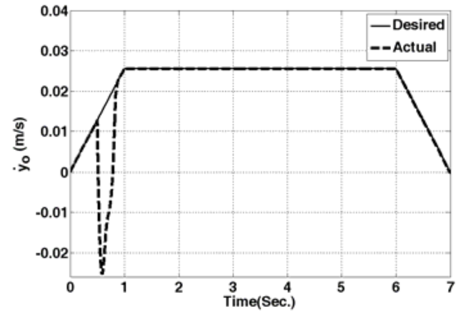

Fig. 7: Vertical velocity tracking of the object.

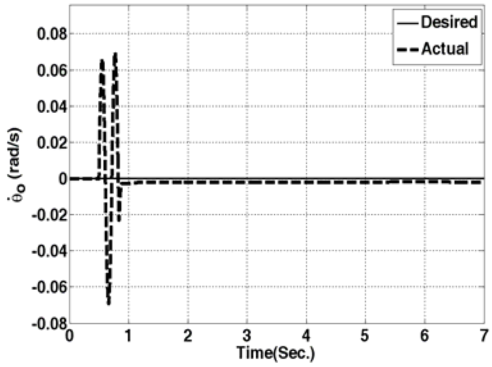

Fig. 8: Rotational velocity tracking of the object.

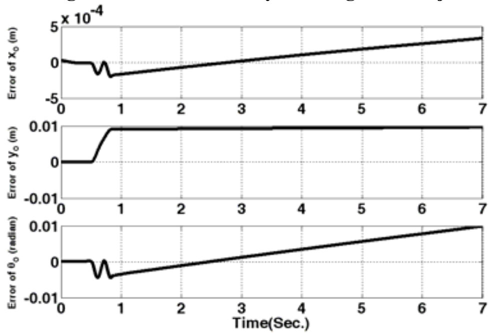

Fig. 9: Error of object position tracking.

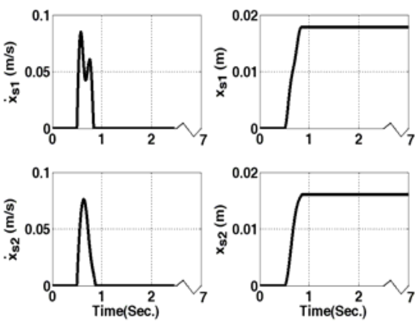

Fig. 10: End-effectors movement on the object surfaces and slippage velocity of the end-effectors.

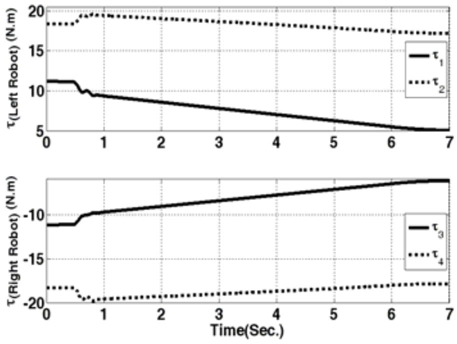

Fig. 11: Time history of manipulators' torques.

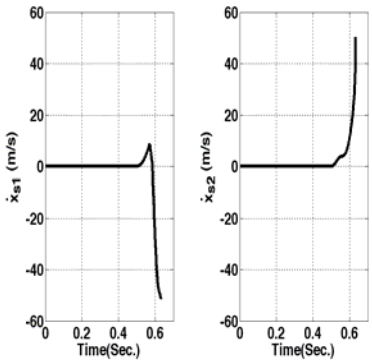

Fig. 12: Slippage velocity of the end-effectors using conventional approach without modification.

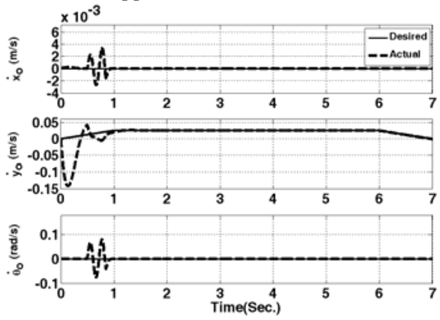

Fig. 13: Object velocity tracking when the model parameters
differ from the actual one.

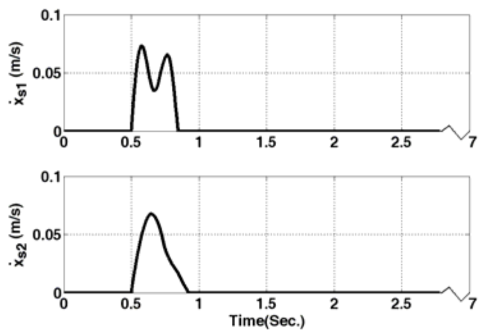

Fig. 14: Sliding velocity when the model parameters differ from the actual one.

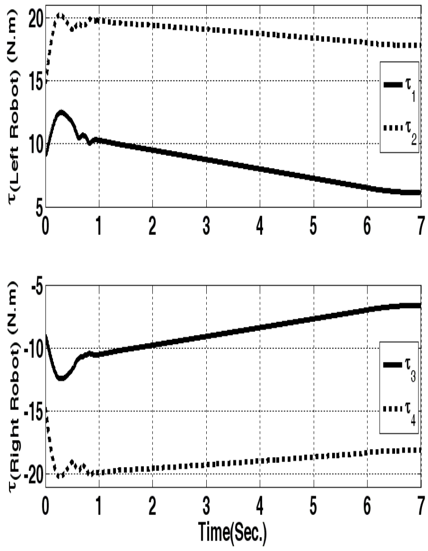

Fig. 15: Time history of manipulators generalized driving forces when the model parameters differ from the actual one.

5-Conclusion

Sliding phenomenon in grasping and manipulation of an object is studied in this paper for a cooperating system with two robot arms. In order to formulate and simulate dynamics of the system, equality and inequality equations of contact conditions are replaced by a single second order differential equation with switching coefficients. Accuracy of this modeling is verified by comparing its results with those of SimMech toolbox of MATLAB. The conventional control method in grasping of an object by a cooperating system is modified for the cases that the end-effector of the manipulator slides on the object surfaces, by including the movement of the end-effector on the object and its velocity in control law. The controller is a hybrid closed loop position controller and an open loop force controller. It was observed that the modified controller can control the sliding and push the sliding velocities converge to zero.

Acknowledgement

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Appendix

A Values of α_i and β_i $(i=1,2,3)$

TABLE A-I

VALUES FOR $\boldsymbol{\alpha}^T=[\alpha_1 \quad \alpha_2 \quad \alpha_3]$ AND $\boldsymbol{\beta}^T=[\beta_1 \quad \beta_2 \quad \beta_3]$ IN DIFFERENT CONDITIONS.

$\dot{x}_{s2}$ \ $\dot{x}_{s1}$		$\dot{x}_{s1}\neq 0$	$\dot{x}_{s1}=0$				
			$\ddot{x}_{s1}^-\neq 0$		$\ddot{x}_{s1}^-=0$		
		Movement	Motion reversing	No Motion	No Motion	Start forward	Start backward
$\dot{x}_{s2}\neq 0$	Movement	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad \text{sign}(\dot{x}_{s2}) \quad 1]$
$\dot{x}_{s2}=0$, $\ddot{x}_{s2}^-\neq 0$	Motion reversing	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 0 \quad 1]$
	No Motion	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$
$\dot{x}_{s2}=0$, $\ddot{x}_{s2}^-=0$	No Motion	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[1 \quad 0 \quad 0]$
	Start forward	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad 1 \quad 1]$
	Start backward	$\boldsymbol{\alpha}^T=[0 \quad -\text{sign}(\dot{x}_{s1}) \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 0 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$	$\boldsymbol{\alpha}^T=[1 \quad 0 \quad 0]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad -1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$	$\boldsymbol{\alpha}^T=[0 \quad 1 \quad 1]$ $\boldsymbol{\beta}^T=[0 \quad -1 \quad 1]$

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