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Optimization of the Kinematic Sensitivity and the Greatest Continuous Circle in the Constant-orientation Workspace of Planar Parallel Mechanisms

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ABSTRACT

This paper presents the results of a comprehensive study on the efficiency of planar parallel mechanisms, considering their kinetostatic performance and also, their workspace. This aim is approached upon proceeding single- and multi-objective optimization procedures. Kinetostatic performances of ten different planar parallel mechanisms are analyzed by resorting to a recent index, kinematic sensitivity. Moreover, the greatest possible continuous circle in the constant-orientation workspace of the latter mechanisms is considered as another objective for the optimization procedures. Seeking the set of design parameters which compromises simultaneous optimal values for the two aforementioned objectives, i.e., kinematic sensitivity and workspace, necessitates launching a multi-objective optimization process. The mathematical framework adopted for the optimization problem is based on genetic algorithm. The results of multi-objective optimization are based on the sets of Pareto points, offering the most reliable decisions to reconciliate between some conflicting objectives. To this end, the ten planar parallel mechanisms are sorted into two sets based on their type of actuator, some of them with prismatic actuators and the other ones with revolute actuators. Finally, a comparison between performances of these mechanisms, according to the obtained results, is carried out.

1. Introduction

Nowadays, robotic mechanical systems are more and more making their way to the industry, and efficient design of them is becoming the main subject of several papers and technical reviews [3–6, 9, 11, 14, 17–19, 21, 22, 29, 31–34]. Various performance indices are introduced and developed for robotic mechanical systems, including among others, parallel mechanisms (PMs). However, most of them entail some drawbacks and are not very communicative for this purpose [9, 10, 12, 13, 16, 23]. For instance, in the case of the kinetostatic performance indices, the units used for the translation and rotation degrees-of-freedom (DOF) are not consistent, therefore the Jacobian matrix is non-homogeneous. Thus, defining an overall index for the performance of the mechanisms will not provide a complete and reliable description and may lead to an erroneous conclusion.

The performance of robotic mechanical systems is combined with the concept of kinematic and static traits, which is tantamount to the so-called kinetostatic properties. In turn, kinetostatic performance index is a scalar quantity that assesses the efficiency of a robot under some uncertainties in the mechanism at the kinematic or static level, where the former and the latter are related by duality [1, 20].

Indeed, kinetostatic analysis in mechanic of rigid bodies is founded by virtue of duality between the kinematic and static relations. The basic concept of kinetostatic studies is the reciprocity between the feasible twist and constraint

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wrench in a rigid body moving under static condition [1, 20]. In what follows, the concept of Jacobian matrix and some main drawbacks of the indices defined in literature, in the context of measuring performance of the PMs are briefly reviewed. Consider $\dot{\boldsymbol{\theta}}$ and $\mathbf{x}$ as small variations in the input and output vectors, respectively.

The first-order kinematic relation for PMs can be written as:

$$0.1 < \mathbf{a}_b < \mathbf{a}_a = 1. \tag{1}$$

where $\mathbf{K}$ is the inverse Jacobian matrix. The two well-known performance indices, dexterity and manipulability [10], are defined on the basis of the above matrix, while the rotational and translational parts of dimensionally non-homogeneous Jacobian matrix are merged together. Therefore, they lead to physically inapplicable interpretations about the mechanisms, as they are normalizing unit-inconsistent quantities. As a consequence, changing the scales of quantities can change the final results and physical interpretations, considerably. This point is asserted and investigated in detail in [23]. Moreover, the characteristic length, proposed in [2], suffers from the as the first step, the single-objective optimization process considers the whole area of the workspace of the PPMs as the criterion. However, the latter quantity refers to an area of the workspace which usually suffers from fragmentations, to a high degree. While trajectory planning, switching between separate regions of the workspace is a demanding, or sometimes even impossible task. As a consequence, the aforementioned optimization strategy is not of practical importance and requires some improvements. Subsequently, another optimization procedure is provided, which considers the greatest possible continuous circle in each orientation of the moving platform of the PPMs, i.e., the constant-orientation workspace, as the objective.

Finally, a Pareto based multi-objective optimization concept, the so-called Non-dominated Sorting Genetic Algorithm II (NSGA-II) [28], is approached and designated to find the optimal design parameters of the PPMs under study, by taking into account point-displacement and rotational kinematic sensitivity, together with the workspace, as the objectives of the optimization problem.

The remainder of this paper is organized as follows. First the geometric modeling and the first-order kinematic analysis, Jacobian matrix of PPMs are reviewed. The paper pursues the study by touching upon some fundamental concept on kinematic sensitivity index. The single- and multi-objective optimization procedures are elaborated where more emphasis is placed on refining the objective parameters to be suitable to be implemented in the optimization procedure. The single-objective procedure is applied by resorting to the DE concept in order to lay down the essentials for the multi-objective optimization. Subsequently, Pareto -based multi-objective optimization is considered and NSGA-II is used to synthesis the optimal mechanism. Finally, the paper concludes with some remarks by putting into contrast the results obtained from the optimization for all the mechanisms.

2. Kinematic Modeling of PPMs

The analytical formulation of kinematic sensitivity of a PPM requires a thorough review of the relations of the first-order kinematic, which is fully elaborated in [7], and its results are broadly reviewed in what follows.

2-1. Classification of the Architectures

From the study conducted in [8], it reveals that there are in total 21 possibilities for generating 3-DOF legs. However, the latter list contains some pairs of kinematically equivalent PPMs. Furthermore, some cases are not able to perform 3-DOF motion, due to the fact that they have just one independent DOF. Therefore, from the latter examination, ten feasible architectures arise, which are credible for the purpose of the study conducted in this paper. Fig. 1 depicts schematically the latter PPMs, in which design parameters are presented for each PPM individually.

All the 10 considered PPMs, compromises 3 identical legs, while just one joint per leg (the underlined one) is actuated. Here and throughout this paper, revolute joints are denoted by R and the prismatic joints by P, where the actuated one is underlined. In this notation, the kinematic arrangement is denoted by writing the consecutive order of the joints from the base to the end-effector, by respecting some symmetric rules among the joints.

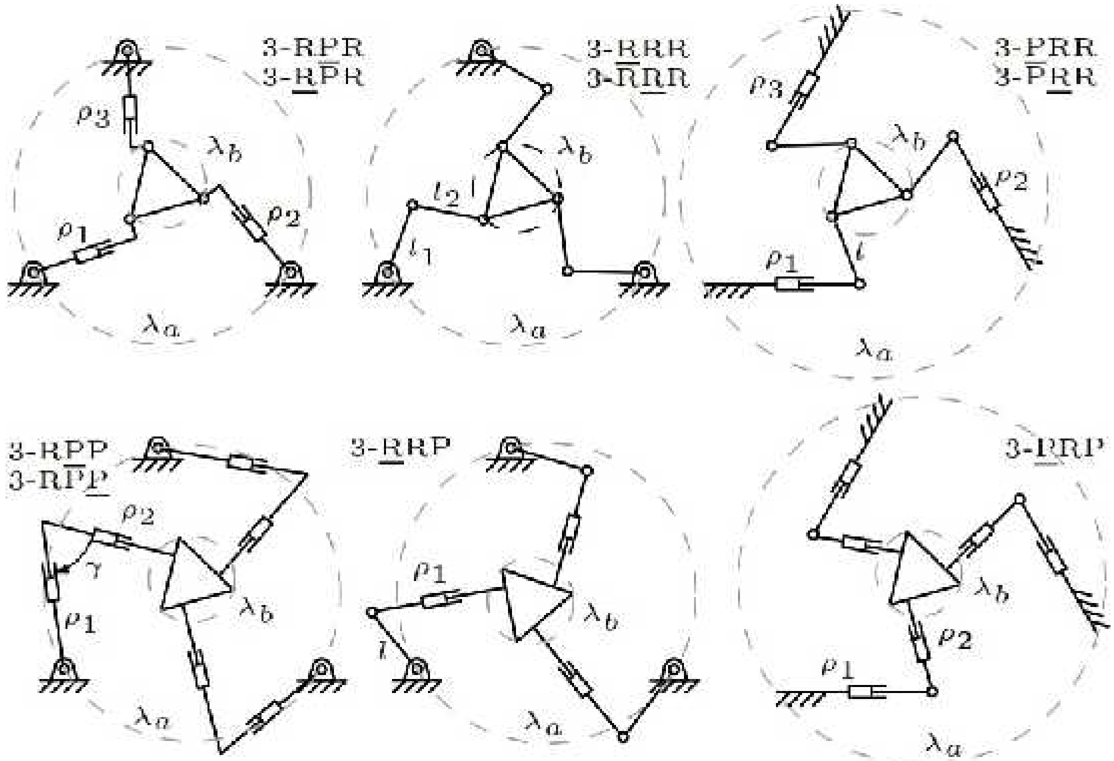


Fig. 1. The 3-DOF PPMs with identical leg structures. The schematic is adopted from [7].

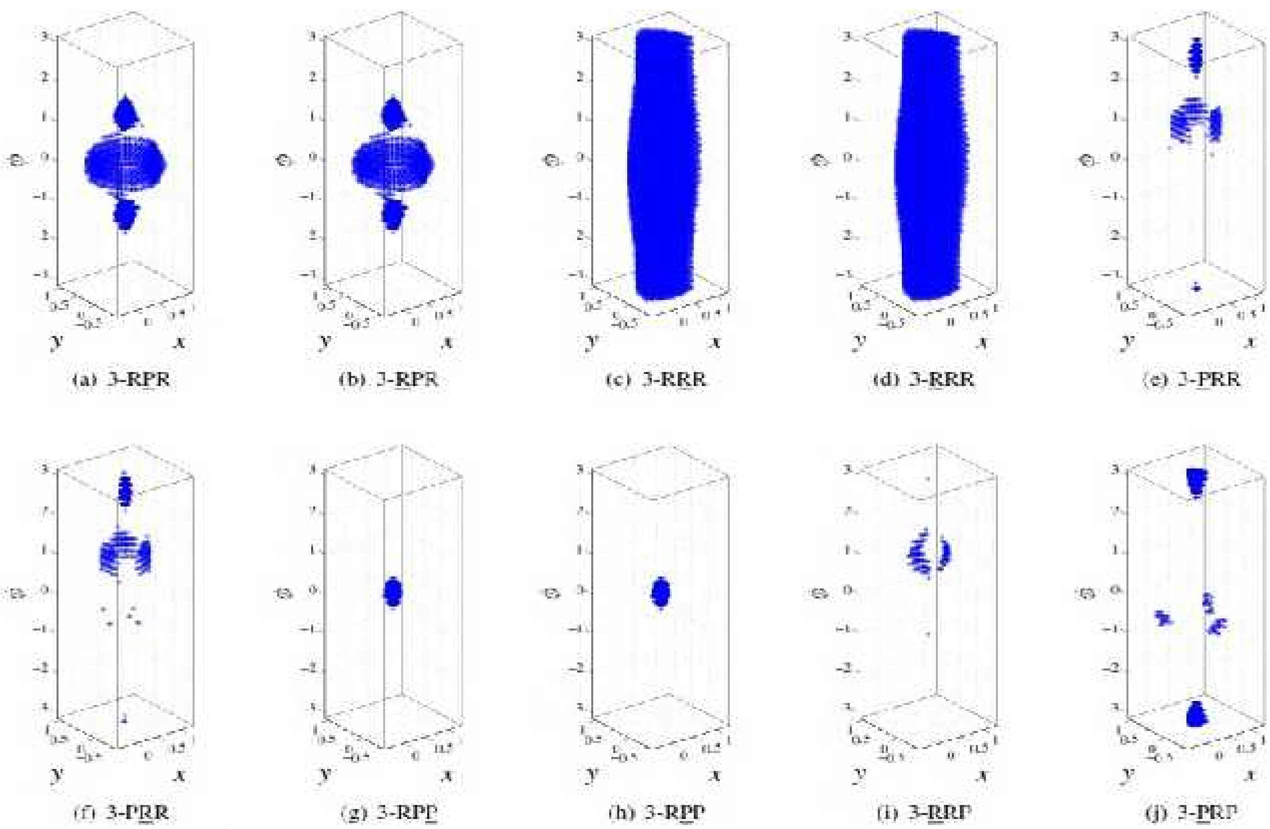


Fig. 2. The workspace of the PPMs obtained from the single-optimization procedure, according to the design parameters shown in Table 1.

Table 1. The results of the single-objective optimization of the greatest continuous circle in each orientation of the moving platform of the PPMs

Result⇒ PPM⇓	Parameters						$\zeta_{\sigma'_{Pos,1}}$	$\zeta_{\sigma'_{Pos,1'}}$	W
$3\text{-}R\underline{P}R$	λ_b 0.1018	$\rho_{\min}$ 0.3684	$\rho_{\max}$ 0.7336	-	-	-	0.4113	0.0409	0.2763
$3\text{-}\underline{R}PR$	λ_b 0.1010	$\rho_{\min}$ 0.3671	$\rho_{\max}$ 0.7309	-	-	-	0.6486	0.0872	0.2763
$3\text{-}R\underline{R}R$	λ_b 0.3899	l_1 0.4994	l_2 0.5000	-	-	-	0.4103	0.1701	4.6486
$3\text{-}\underline{R}RR$	λ_b 0.3890	l_1 0.4997	l_2 0.4991	-	-	-	0.3988	0.1742	4.6486
$3\text{-}\underline{P}RR$	λ_b 0.9729	$\rho_{\min}$ 0.2965	$\rho_{\max}$ 0.5897	l 0.4782	-	-	0.5674	0.3954	0.0197
$3\text{-}P\underline{R}R$	λ_b 0.9154	$\rho_{\min}$ 0.2928	$\rho_{\max}$ 0.5855	l 0.4929	-	-	0.9047	0.7073	0.0197
$3\text{-}RP\underline{P}$	λ_b 0.6134	$\rho_{1\min}$ 0.3775	$\rho_{1\max}$ 0.7230	$\rho_{2\min}$ 0.3482	$\rho_{2\max}$ 0.6842	γ 0.9405	0.6722	0.4726	0.0296
$3\text{-}R\underline{P}P$	λ_b 0.3316	$\rho_{1\min}$ 0.3985	$\rho_{1\max}$ 0.7378	$\rho_{2\min}$ 0.3640	$\rho_{2\max}$ 0.7223	γ 0.9474	0.6292	0.2442	0.0197

2.2. First-order Kinematic Relations, Jacobian Matrix

From a geometrical standpoint, the Jacobian matrix can be regarded as a mapping of a unite-ball in the joint rate space into a deformed (rotated and reflected) ellipsoid in the Cartesian velocity space, known as the twist of the end -effector [1, 20]. For our purpose, the inverse Jacobian matrix can be interpreted as a linear transformation which maps a finite error in the joint space into the Cartesian space of the mechanism [27].

The latter matrix can be exemplified by referring to the kinetostatic performance indices proposed in the literature where almost all of them are based, by different perspectives, on Jacobian matrix. As fully described in [8], the input-output velocity equation can be expressed as follows:

$$\begin{bmatrix} \zeta_1^\circ \\ \zeta_2^\circ \\ \zeta_3^\circ \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \mu \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{a}_3 \end{bmatrix}, \tag{2}$$

or in a matrix form:

$$\mathbf{Z}\boldsymbol{\xi} = \boldsymbol{\Lambda}\dot{\boldsymbol{\theta}} \Rightarrow \mathbf{K}\boldsymbol{\xi} = \dot{\boldsymbol{\theta}}, \tag{3}$$

where $\mathbf{K} = \boldsymbol{\Lambda}^{-1}\mathbf{Z}$ stands for the inverse Jacobian matrix. In the relation above, ζ_i° is the row vector of the three-dimensional matrix of wrench, implied by the i^{th}. Furthermore, in the above, a_i is the moment of the reciprocal force with respect to the center of the active joint, where the actuator is revolute, or the projection of the force onto the direction of the actuated translation where the actuator is prismatic. It should be noted that i is a dummy variable, which is subject to be changed throughout the paper.

As mentioned previously, kinematic sensitivity is considered for the purpose of this paper. In order to find a single scalar value describing the kinematic sensitivity of the whole workspace, the average kinematic sensitivity of the mechanism, referred to as global kinematic sensitivity, is considered. For this purpose, it should be recognized that which points are within the workspace of the mechanism for any given orientation. Thus global kinematic sensitivity can be regarded as an index representing the performance of the mechanism for the feasible workspace.

2.3. Workspace Determination

Workspace of robotic mechanical systems is investigated upon different perspectives. Among several types proposed in the literature, in this paper, the most common one is considered, known as constant-orientation workspace. Constant-orientation workspace is the set of all possible Cartesian coordinates of a given point of the mobile platform that can be reached for a prescribed orientation. The volume of the workspace can be calculated numerically using discrete integration, which can be formulated as follows [28]:

$$W = \int_{-®}^{®} A(¤)\,d¤, \tag{4}$$

where $A(¤)$ is the area of the constant-orientation workspace. In this paper, the Inverse Kinematic Problem (IKP) is solved for all of the possible positions and orientations (poses) of the moving platform of the mechanisms to obtain the volume of the workspace. For each pose in the Cartesian space, (x, y, ϕ), if the solution of the IKP verifies the conditions and constraints of all joints, i.e., the stroke of the actuators, this point is a valid point and lies within the workspace.

2.4. Global Kinematic Sensitivity

From [10], using a geometric standpoint, kinematic sensitivity is defined as the maximum displacement/rotation of the moving platform of the mechanism, under a unit norm displacement/rotation in the joint space. The components of the Jacobian matrix for a general PM performing both position and orientation tasks are dimensionally non-homogeneous. In order to overcome the latter problem, from the definition given in [10], two different types of kinematic sensitivities, namely point-displacement kinematic sensitivity and rotational point-displacement kinematic sensitivity and rotational kinematic sensitivity are introduced, which can be expressed mathematically as:

$$\sigma_{r_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c = 1} \|\phi\|_f , \qquad \sigma_{p_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c = 1} \|\mathbf{p}\|_f , \tag{5}$$

where $\sigma_{r_{c,f}}$ and $\sigma_{p_{c,f}}$ are the rotational and point-displacement kinematic sensitivity, respectively. Moreover, c and f are the norms of the constraints function and the norm of the pose of the moving platform, respectively. In the above, the vector $\mathbf{p}$ and the scalar ϕ show variation of position and orientation of the moving platform, respectively.

The most relevant norms which are arising more in theoretical and practical problems are the 2- and ∞-norm. Therefore, four combinations are possible to define kinematic sensitivity. From the comprehensive study conducted in [23, 27], it reveals that $c = \infty$ and $f=2$ bears the most meaningful and self-evident representation, and thus, is considered for the aim of this paper.

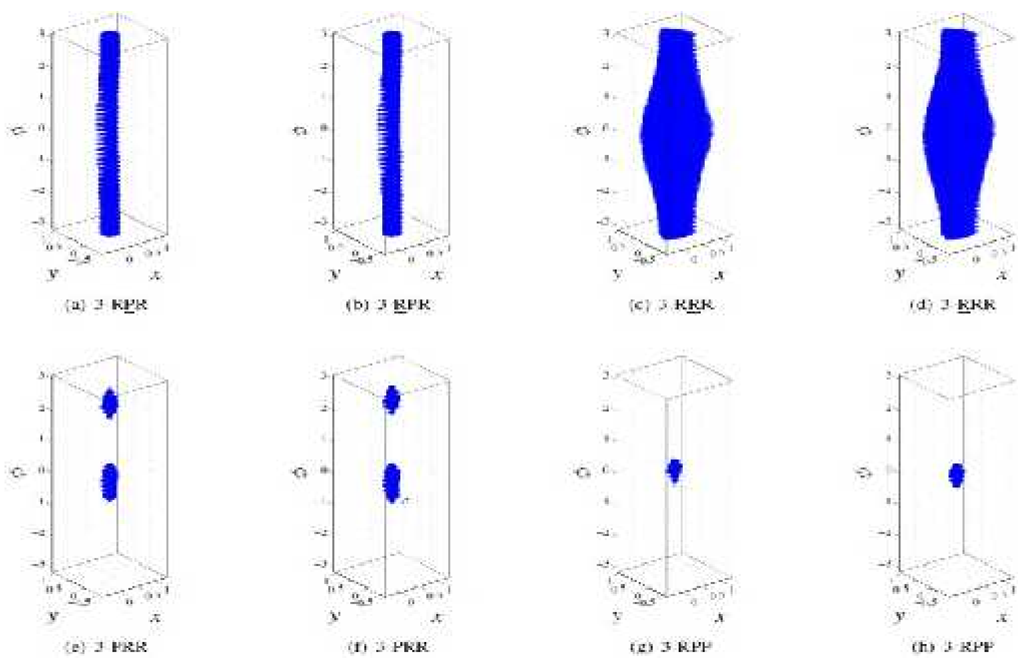


Fig. 3. The whole workspace, after the single-objective optimization of the greatest continuous circle in each orientation of the PPMs

According to Eqs. (3) and (5), the constraint $\|\dot{\boldsymbol{\theta}}\|_c \le 1$ for $c = \infty$ can be rewritten as follows:

$$\|\mathbf{Kx}\|_\infty \le 1. \tag{6}$$

The above can be made equivalent geometrically to a polyhedron spanned by 2^n corners. Moreover, $\mathbf{x} = [x \quad y \quad \phi]^T$ represents the pose of the moving platform.

In short, from the above reasoning, the problem of kinematic sensitivity can be streamlined by obtaining the corners of the latter polyhedron. To do so, one should solve the following system of equations:

$$\mathbf{L}\Delta\mathbf{x} \prec \mathbf{1}_{2n}, \qquad \mathbf{L} \equiv \begin{bmatrix} \mathbf{K}^T & -\mathbf{K}^T \end{bmatrix}^T, \tag{7}$$

where the sign $\prec$ denotes for the component-wise inequality. Since the polyhedron is symmetric with respect to the origin, thus solving 2^{n-1} equations from the above system of equations is sufficient to obtain all the corners. In this case, since $n=3$, thus the polyhedron has eight corners, which leads to eight inequalities. According to the above reasoning, only four inequalities should be solved, which leads to four corners of the corresponding polyhedron, denoted as (x_i, y_i), $i = 1, \ldots, 4$. Finally, for $c = \infty$ and f=2, the point-displacement kinematic sensitivity, $\pm_{p_{\infty,2}}$, and rotational kinematic sensitivity, $\pm_{r_{\infty,2}}$, can be formulated as follows [26, 27]:

$$\pm_{p_{\infty,2}} = \max_{i=1,\ldots,4} \sqrt{x_i^2 + y_i^2}, \qquad \pm_{r_{\infty,2}} = \max_{i=1,\ldots,4} ¤_i . \tag{8}$$

Having in the mind that the above are nonnegative quantities, then considering

$$\pm'_{p_{\infty,2}} = \frac{1}{1+\pm_{p_{\infty,2}}}, \qquad \pm'_{r_{\infty,2}} = \frac{1}{1+\pm_{r_{\infty,2}}}, \tag{9}$$

leads to bound them between 0 and 1, a maneuver akin to the one proposed in [28]. These values give a more tangible insight for the performance of the mechanism and after averaging in the whole workspace one can obtain the global kinematic sensitivity of the mechanism (as described in [25, 28]), as follows:

$$¸_{\pm'_{p_{\infty,2}}} = \frac{\int_W \pm'_{p_{\infty,2}} dW}{\int_W dW}, \qquad ¸_{\pm'_{r_{\infty,2}}} = \frac{\int_W \pm'_{r_{\infty,2}} dW}{\int_W dW}. \tag{10}$$

3. Optimization Procedures

First and foremost, an optimization problem requires a suitable representation for its objectives (goals) and parameters. In the case of this paper, as pointed out previously, the objectives consist in minimizing the global kinematic sensitivity, which is equal to maximizing Eq. (10), while keeping the workspace of the mechanism as great as possible. Parameters which are subject to be optimized are the geometric parameters of the architectures represented in Fig. 1. In order to make a fair comparison for all the ten PPMs, the three fixed points attached to the base are assumed to lie on a circle with $ª_a = 1$ as diameter and form an equilateral triangle. Other design parameters should first be bounded to a rational range, to satisfy the conditions and requisitions of practical work, such as the stroke of actuators and reasonable scale between the base and moving platform. It should be noted that the design parameters are assumed to be equal for all of the three legs and hence to be considered in the optimization procedure as only one set of parameters. The task of optimization can be implemented as the process of objection of one or more of the desired goals. The former usually gives unreasonable results as a consequence of the fact that the procedure is devoting the other objectives to pleasing the one selected, at the greatest extent. More precisely, as it is explained in detail in the next sections, maximization of the global kinematic sensitivity index, leads to terrible results for the workspace, in the most cases. Therefore a multi-objective optimization should be launched to obtain a set of most possibly reliable solutions for all the objectives. However, the results obtained from single-objective optimization are of great importance to have an initial insight to design of the mechanism and to use the initial values for the multi-objective optimization, in spite of its low credibility for practical purposes.

In what follows, first the above mentioned ranges and conditions on the parameters are given and then the concepts and results of single- and multi-objective optimization procedure are illustrated. It should be noted that, according to some limitations on the prepared packages provided by computer algebra systems, the software codes are developed by the authors.

3.1. The Initial Design Constraints

As mentioned above, in order to avoid lower bound solutions, some of the design parameters should be limited to a reasonable range, before launching the optimization procedure. In the following, some of these constraints and ranges are briefly reviewed. Due to the stroke of the prismatic joints, they are mechanically restricted to have minimum and maximum stroke as $°_{\min} > 0.1$ and $°_{\max} < 0.75$. Thus, it can be readily deduced that:

$$°_{\min} > 0.1, \qquad °_{\max} < 0.75, \qquad °_{\max} > °_{\min}, \tag{11}$$

and:

$$°_{\max} - °_{\min} < °_{\min} \Rightarrow °_{\max} < 2°_{\min}. \tag{12}$$

Since mechanical interferences are ignored in this research, thus there is no limitation for the revolute joints and they are considered to be able to take any requisite value between 0 and 2®. For all the ten mechanisms, the three connection points on the moving platform lie on a circle with $ª_b$ as radius and form an equilateral triangle. According to the fact that the moving platform should have reasonable size with respect to the base, to carry the device, thus the circle defined above with $ª_b$ as radius should be bound follows:

$$0.1 < ª_b < ª_a = 1. \tag{13}$$

In order to meet practical criteria, rigid links are assumed to satisfy the condition $0.05 < l < 0.5$.

3.2. Single-objective Optimization Using Differential Evolution

The results of the single-objective optimization are obtained by following and implementing the concept of DE explained in [28], in a computer algebra system. As mentioned before, these results are of less concern in practice, since they devote the other performance indices to pleasing the objected one to the highest extent. The obtained results reveal that, in most of the cases, optimizing one of the objectives, leads to an irrational worsening of one or two of the other objectives. For example, optimizing one of the kinematic sensitivity indices, results in a very limited (sometimes bounded to only a point) workspace, which excludes the resulted design of any practical importance.

Moreover, single-objective optimization of the workspace of the PPMs usually leads to wide, though non-continuous, workspaces. As aforementioned, according to the fact that, while trajectory planning, switching between different regions of the workspace is a demanding, or even sometimes impossible task, this attitude for optimization, as well, does not provide practical applicability. The workspaces of all the ten PPMs, resulted from single-objective optimization of the workspace, are depicted in Fig. 2, for the sake of illustration.

In order to circumvent the non-continuousness of the workspaces of the PPMs, another perspective is provided for the single-objective optimization procedure, which searches for the greatest possible continuous circle in each orientation of the moving platform of the PPMs, i.e., their constant- orientation workspace, and considers the latter parameter as the objective of the optimization process.

The process of finding the greatest circle in the constant-orientation workspace of the PPMs starts at the origin of the Cartesian space, and proceeds with increasing the radius of the circle, while checking for any fragmentation, which leads to ending up the process. More precisely, it commences the computation process by considering a circle with the lowest radius and checking whether all the points within the circle are included in the constant-orientation workspace of the PPM. If any point is not included, the process stops, and moves on to be repeated for other orientations. Otherwise, i.e., if all the points are included in the workspace, then the radius will take a one-step larger value, and the process will be reiterated. The foregoing procedure goes on until the largest circle, which obviously corresponds to the largest radius, is obtained and subsequently, the greatest circles associated with all possible orientations of the moving framework are aggregately taken into account as the whole workspace of the PPM.

The parameters obtained from the aforementioned optimization procedure are shown in Table I, along with the resulted value of each objective in each case. It should be noted that the 3-RRP and the 3-PRP PPMs have very limited workspaces, especially near the origin, which is the point of concentration of the aforementioned optimization procedure. Therefore, the latter two PPMs are excluded from the rest of the table. Furthermore, the whole workspace of the all eight remaining PPMs (including the points outside the optimized circle in each cross-section of their workspace, as well) are shown in Fig. 3, in order to put them into contrast with the workspace of the previous designs, depicted in Fig. 2.

It should be noted that the designs obtained from the latter single-objective optimization procedure, provide wide continuous workspaces. By the way, they suffer from heavy singular regions, and moreover, present terrible error amplification, sought from their associated kinematic sensitivity. Although the single-objective optimization procedure has no immediate feasible result, it provides very helpful consequences for providing some insight about practical and reasonable values for design parameters. According to Table I, it follows that:

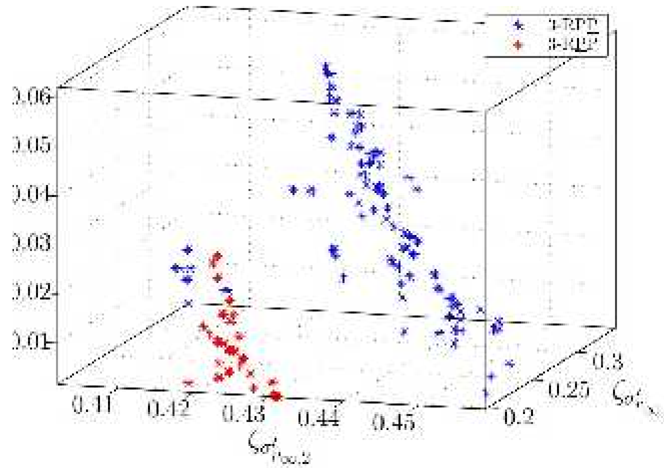


Fig. 5. Pareto points resulted from multi-objective optimization for 3-RPP and 3-RPP PPMs.

- Comparison of 3-RPR and 3-RPR PPMs: These two mechanisms get nearly the same optimal workspace, but the latter seems to have a better kinematic sensitivity.
- Comparison of 3-RRR and 3-RRR PPMs: These two mechanisms do not seem to have any notable difference in none of the optimization objectives.
- Comparison of 3-PRR and 3-PRR PPMs: These two mechanisms get nearly the same optimal workspace, but the latter seems to have a better kinematic sensitivity.
- Comparison of 3-RPP and 3-RPP PPMs: These two mechanisms get nearly the same optimal workspace, but the former seems to have a better kinematic sensitivity.

From the above it can be concluded that PPMs with revolute actuators seem to have better performance than PPMs with prismatic actuators, in terms of kinematic sensitivity, which was sought from the outset.

4. Multi-objective Optimization Using NSGA-II

Naturally, optimization of PPMs, objecting one of kinetostatic performance indices, such as kinematic sensitivity, may lead to both lowering the error amplification of the PPM and decreasing the area of the singular regions in their workspace. However, as aforementioned, single objective optimization procedures usually do not provide practicable results, which has urged to perform multi-objective processes, which should satisfy both of the conflicting objectives, i.e., kinematic sensitivity and workspace, simultaneously.

In order to perform a comparison, the compared values of the obtained performance indices must be unit-consistent.

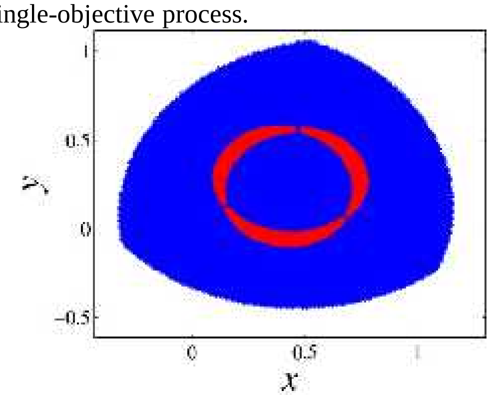
About mechanisms with revolute actuators, the unit of point-displacement kinematic sensitivity is m/rad and their rotational kinematic sensitivity is unit-less; while for mechanisms with prismatic actuators, the point-displacement kinematic sensitivity is unit-less and the unit of the rotational kinematic sensitivity is rad/m. This leads to reclassify the ten PPMs into two sets: with revolute actuators and with prismatic actuators.

The Pareto points resulted from this procedure, are illustrated for mechanisms with revolute actuators in Fig. 4. From the latter figure, it can be concluded that 3-RRR and 3-RRR PPMs have better performance by taking into account all the three objectives. However, discussion and comparison of the mechanisms necessitates defining appropriate decision makers, which is beyond the scope and length of this paper.

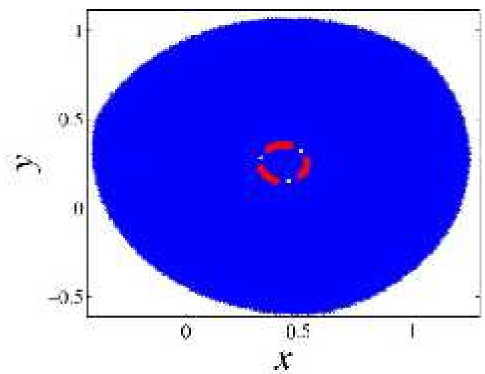
Among mechanisms with active prismatic joint, Paretos points related to the 3-RPP and the 3-RPP PPMs are shown in Fig. 5. It is clear from the latter figure that the Pareto for the 3-RPP PPMs dominates the one for the 3-RPP PPMs. Thus, it can be inferred that the 3-RPP mechanism provides a better performance than the 3-RPP, from both workspace and kinematic sensitivity points of view.

According to the fact that the multi-objective optimization procedure reconciles between the kinematic sensitivity and workspace, basically, the choice of a design with a wider range of workspace, results in a poorer kinematic sensitivity, i.e., more error amplification. Consequently, the user or designer of the PPM, should select one of the designs indicated by Pareto points, upon their own preference of the kinematic sensitivity and workspace.

For example, Fig. 6 presents the constant-orientation workspace and singular regions of the one of the designs provided within the sets of Pareto points for the 3-RRR PPM, in $\alpha = 10^\circ$, together with the corresponding previous design, obtained from the single-objective procedure. Moreover, Fig. 7 provides the same illustration, for the point-displacement kinematic sensitivity. The results confirm that the singularity locus of the PPM has reduced by the multi-objective optimization procedure, although, naturally, its workspace is not as wide of the designs obtained from the single-objective process.

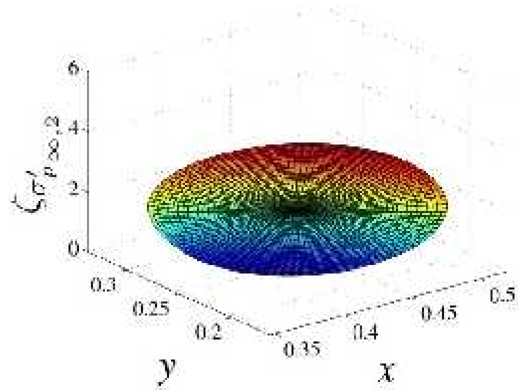


(a) After single objective optimization

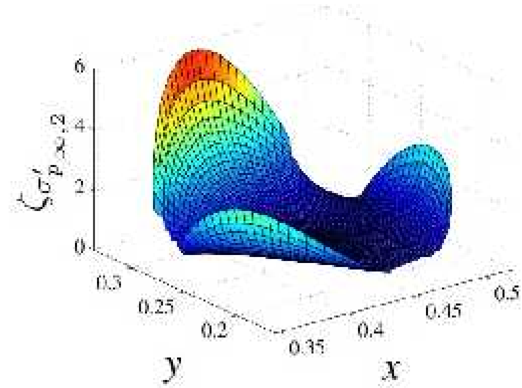


(b) After multi objective optimization

Fig. 6. The constant-orientation workspace (blue points) and the singular areas (red points) of the 3-RRR PPM.



(a) After single-objective optimization



(b) After multi-objective optimization

Fig. 7. The point-displacement kinematic sensitivity of the 3-RRR PPM.

5. Conclusion

This paper investigated the kinetostatic performance of the planar parallel mechanisms. Their geometrical dimensions were subject to be optimized based on their performance criteria, including point-displacement and rotational kinematic sensitivity and also workspace. The optimization procedure considered the greatest continuous circle in each orientation of the moving platform of the mechanisms as one of the criteria. First, a single-objective optimization was launched, using an evolutionary technique, the so-called differential evolution. The aim of this procedure was to provide an initial insight into the effect of design parameter on each of the optimization objectives, individually. Based on the results of the single-objective optimization, it was concluded that the $3\text{-}\underline{R}RR$ and $3\text{-}R\underline{R}R$ planar parallel mechanisms have better performance from both workspace and kinematic sensitivity points of view. Also it was concluded that the $3\text{-}P\underline{R}R$ has an acceptable kinematic sensitivity with respect to the other planar parallel mechanisms. Finally, a multi-objective optimization was approached, using non-dominated sorting genetic algorithm-II, whose results were presented upon some sets of Pareto points. The results of the multi-objective optimization revealed that the Pareto points for the $3\text{-}R\underline{R}R$ and $3\text{-}\underline{R}RR$ dominate the ones for the other planar parallel mechanisms with revolute actuators, as well as the Pareto points for the $3\text{-}RP\underline{P}$ dominate the ones for the $3\text{-}R\underline{P}P$, which is readily a sign of the better performance of the former, with respect to the all of the three objectives. Moreover, a brief comparison between the results of the single- and multi-objective optimization procedures was provided, which proved the credibility and feasibility of the introduced optimization algorithm. Ongoing works include the optimum synthesis of more complex parallel mechanisms and implementing an appropriate decision-maker algorithm for refining the Pareto results.

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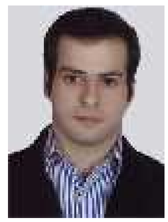

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