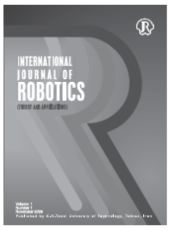

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α -Stabilized Optimal Controller for Feedback Linearized Feedback-Decoupled Quaternion Based Model of Stewart Platform

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ABSTRACT

Here, we are going to design a stabilized controller for the Stewart platform. A new model of the Stewart platform will be presented which has many applications in the industry. The dynamics of the Stewart platform are presented in two separate systems. One system for the linear motion of the Stewart platform and another system for its angular moment. In addition, we use a quaternion-based method to analyze the dynamics of the Stewart platform. The 6-DOF Stewart-Platform dynamics is nonlinear, then at first by using the feedback linearization method we convert the nonlinear dynamics in new space as a linear state-space. Then we design an α -stabilized controller for this platform in linear space. A linear controller for linear motion systems will be designed but for the second system, it must first be linearization and then design controller for it. After design of stabilized-LQR controller for linearized space system, we convert our design to original nonlinear space and exert on system for simulations. The simulations results show that we will succeed to design a controller for the Stewart platform.

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1. Introduction

Stewart platform is a parallel robot that employs a closed-loop kinematic chain (CKC). The platform was at the first made in 1965 to simulator aircraft [1]. In particular, when compared to open–loop kinematic chain mechanisms, CKC manipulators have a higher structural rigidity, noncumulative actuator errors, and a proportionally distributed payload to the links, granting a higher strength-to-weight ratio [2]. Therefore, there is significant interest in parallel manipulators in general and in the 6-DOF Stewart platform in particular [3], [4], [5], whose modern applications range from industrial-grade manipulators [6] to offshore cargo transfer mechanisms [7]. The new model of the Stewart platform that is presented in this article has 6 rotary servos. This six-servo motor will connect the base to the top of the Stewart platform with 6 fixed length bars. The joints between the bar and top platform and servo arms and bars should be universal, so as to move in six spatial positions fulfilled. With this scheme, we will able to provide more applications in various industries of this robot. For example, the surgical robot in the medical industry and packaging robots named and etc. Recently, Iranian engineers in Tarbiat Modares University recently build and tested successfully a mobile based Stewart platform [8]. Meanwhile in another Iranian journal we reported the sensivity analysis of this platform with respect to parameter variations [9]. An optimal control methods for control of this robot is reported in [10] and also an optimization-based fuzzy controller are developed in [11].

This paper is organized as follows: in the first section (section3), at first write the rigid-body dynamics of parallel Stewart manipulator and then re-formulize this dynamics using quaternion method. In section4, we using the famous linearization method from nonlinear control systems, i.e. feedback-linearization to convert nonlinear dynamic quaternion model to a linear state-space model. The benefit of linear state-space model is that the design of control in this space is simpler and are developed many years ago by control engineers. In last part of this section we design a stabilized linear quadratic regularor for linearized system and after design in this space, we return the designed state-feedback controller and the parallel robot manipulator to original nonlinear-space for simulation in last part of the section. Finally conclusion results and we give some proposals for future works.

2. Nomenclature

Here we introduce the symbols that are used in this paper. They are reported in the following table.

Table 1 Nomenclature of this paper

Symbol	Definition	Symbol	Definition
ω	Angular velocity	v	Linear velocity
I_m	Inertia tensor	R	Rotation matrix
q	Orientation	J	Jacobian matrix
p	Position vector	x	State vector
ρ	Quaternion	u	Control vector
τ	Torque vector	y	Output vector
F	Force vector	K	Feedback gain

3. Quaternion-based Description of the Stewart Platform

Stewart platform is a 6-DOF parallel manipulator that consists static base and a movable platform which are linked by six variable-length actuators, as depicted in Figure 1.

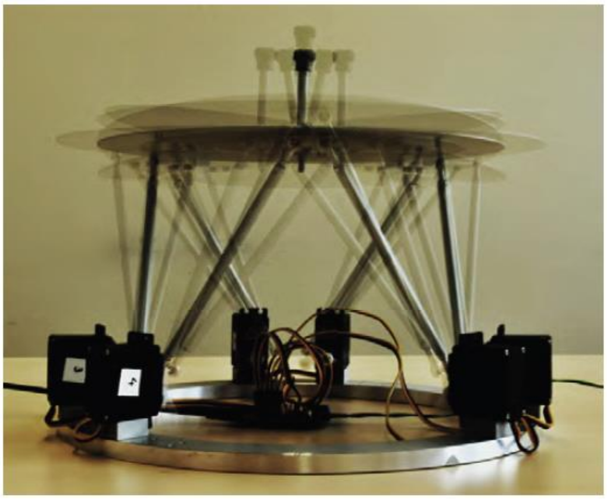

Figure 1. Stewart platform

3.1. Rigid Body Dynamics

In [12] the dynamic model in the joint space of Stewart platform is expressed by:

$$\ddot{q} = J_P \ddot{x}_P + U_P \;\; or \;\; \ddot{x}_P = J_P^{-1}(\ddot{q} - U_P)$$
$$U_P = \begin{bmatrix} (\omega_P \times s_1)(\omega_P \times b_1) \\ \vdots \\ (\omega_P \times s_6)(\omega_P \times b_6) \end{bmatrix} \quad (1)$$

As you see solving this dynamic equation is very difficult and we are going to represent a new method for solving this equation. So let's take a look at Newton-Euler equation of the paper model. The platform is a non-linear coupled system usually modeled using Newton-Euler formalism. By using the classic description of a 3D rigid body with respect to a coordinate frame whose origin coincides with the center of mass of the body, according to [13], the Newton-Euler equations that represent the upper platform are given by:

$$\begin{aligned} \tau &= I_m \omega + S(\omega) I_m \omega \\ F &= m\dot{v} \end{aligned} \quad (2)$$

Here $\tau \in \Re^3$ is the torque vector, $I_m \in \Re^{3\times3}$ is the inertia tensor, and $\omega \in \Re^3$ is the angular velocity vector, all represented in the local body frame of the upper platform. Also, $F \in \Re^3$ is the force vector and $v \in \Re^3$ is the linear velocity vector, where these last two are represented in the global inertial frame, and *m* is the body mass of the end effector, whose center of mass is described point in **Error! Reference source not found.**. The term $S(\omega) I_m \omega$ represents the gyroscopic effect on the platform[2]. In order to relate the dynamics of the velocities, position and orientation of the upper platform, the following mapping will be used,

$$\dot{q} = \frac{1}{2} \begin{bmatrix} -\rho^T \\ aI + S(\rho) \end{bmatrix} \omega \quad (3)$$

Here, $q = \begin{bmatrix} a & \rho^T \end{bmatrix} \in \Re^4, \rho = \begin{bmatrix} b & c & d \end{bmatrix}$ is the body orientation unit quaternion [13] and $p = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T \in \Re^3$ is the position vector of the end effector regarding the global inertial frame, with p_x, p_y and p_z related to the *x*-, *y* and *z*-axis respectively.

Adding the gravity force on the system, the complete dynamics of the upper platform can then be expressed by:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{\omega} \\ \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \begin{bmatrix} -\rho^T \\ aI + S(\rho) \end{bmatrix} \omega \\ I_m^{-1}(u_\tau - S(\omega) I_m \omega) \\ m^{-1}\overline{u}_F + g \\ \end{bmatrix} \quad (4)$$

where $u_\tau \in \Re^3$ is the input torque referenced to the local body frame, $\overline{u}_F \in \Re^3$ is the input force referenced on the global inertial frame and g is the gravity vector.

3.2. Quaternion-based Jacobian

As a standard definition, the Jacobian matrix J transforms the linear velocities of the six actuators $\dot{l} \in \Re^6$ to the linear and angular velocities of the platform, $\dot{p}_j \in \Re^3$ and $\omega_j \in \Re^3$, respectively, with J=(T,B). That is,

$$\dot{l} = J \begin{bmatrix} \dot{P}_T & \omega_T & \dot{P}_B & \omega_B \end{bmatrix} \quad (5)$$

with angular velocities ω_T and ω_B referenced in each local body frame. Going further, from *Figure 2,* the vectors T_i and B_i are defined from the center of the top and bottom platforms, to the i^{th} top and bottom links, relative to the top and bottom platforms, respectively, and P_T, P_B are the position vectors of the platforms.

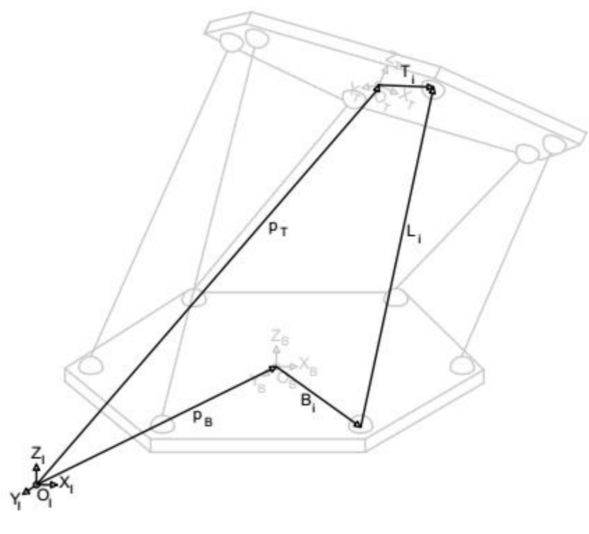

Figure 2 Main vectors of the platform

Consider the Stewart platform and vectors shown on *Figure 2*. There exists a Jacobian matrix $J \in \Re^{6\times12}$ given by

[2] In (1) and in further equations, the skew-symmetric matrix $S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ x_2 & x_1 & 0 \end{bmatrix}, \forall x \in \Re^3$ is used to represent the vector cross product.

$$j=\begin{bmatrix} n_1^T & (S(R_T^I T_1)n_1)^T & -n_1^T & (S(R_B^I B_1)n_1)^T \\ \vdots & \vdots & \vdots & \vdots \\ n_6^T & (S(R_T^I T_6)n_6)^T & -n_6^T & (S(R_B^I B_6)n_6)^T \end{bmatrix} \quad (6)$$

if the relation (5) holds true, where

$$n_i = \frac{L_i}{\|L_i\|} \quad (7)$$

is an unit vector with the same direction of the i^{th} leg $L_i, (i=1,...,6)$ such as

$$L_i = R_T^I + p^T - (R_B^I B_i + p^B) \quad (8)$$

and R_T^I and R_B^I are the rotation matrices regarding the global inertial frame of the platforms, given by

$$R_j(a_j,\rho_j) = I + 2a_j S(\rho_j) + 2S^2(\rho_j) \quad (9)$$

where a_j and ρ_j are the real and imaginary values of $q_j, j=T,B$. If the global inertial frame coincides with the local reference frame of the bottom platform of the manipulator, (8) is simplified to

$$L_i = R_T^I + p^T \quad (10)$$

and the Jacobian matrix (6) is given by

$$J=\begin{bmatrix} n_1^T & (S(R_T^I T_1)n_1)^T \\ \vdots & \vdots \\ n_6^T & (S(R_T^I T_6)n_6)^T \end{bmatrix} \quad (11)$$

Another important result used in this work is that use of the Jacobian matrix to relate the forces and torques of the platform to the forces of the six actuators that power the manipulator.

If J is a Jacobian matrix, its transpose J^T may also be used to relate the linear forces of the six actuators $f_l = [fl_1 \quad \cdots \quad fl_6]$ to the forces and torques applied on the top (F_T and τ_T) and bottom platforms (F_B and τ_B), that is

$$F=\begin{bmatrix} F_T \\ \tau_T \\ F_B \\ \tau_B \end{bmatrix} = J^T f_l \quad (12)$$

4. Control Strategies

According to the (4) we can extract two systems for Stewart platform. Once for linear motion of platform ($\bar{S}_1$) and another for angular motion of Stewart platform ($\bar{S}_2$). So we have

$$\bar{S}_1 = \begin{cases} \begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ m^{-1}I \end{bmatrix} u_\tau \\ x_1 = \begin{bmatrix} p \\ v \end{bmatrix} \end{cases} \quad (13)$$

Where $u_F = \bar{u}_F - g$ and:

$$\bar{S}_2 = \begin{cases} \begin{bmatrix} \dot{q} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\begin{bmatrix} -\rho^T \\ aI + S(\rho) \end{bmatrix}\omega \\ I_m^{-1}(u_\tau - S(\omega) I_m \omega) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_m^{-1} \end{bmatrix} u_\tau \\ x = \begin{bmatrix} q \\ \omega \end{bmatrix} \end{cases} \quad (14)$$

Therefore, our aim in this section is design two separate controllers for each of these systems. First design an LQR controller for the linear system ($\bar{S}_1$) and then controller design for the angular system ($\bar{S}_2$) will be discussed later. Because this system is a nonlinear system, first of all linearization operations will be done for the system and then controller design is done.

4.1. α-Stabilized Controller

We consider a continuous-time linear system described by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (15)$$

In which y is the output of the system and C is the output matrix. Since we are designing a state-feedback system while keeping in mind that all state variables are available for measurement, y can consist of all six state variables and C can thus be an identity matrix. The performance index J that must be minimized to achieve α-stabilized controller for the above linear system is as follows [14-15]:

$$\cos t = \int_0^\infty e^{-2at}\left(x^T Q x + u^T R u + 2x^T N u\right) dt \quad (16)$$

In which x is a vector of state variables, u is a vector of system inputs, and Q,R,N are weighting matrices chosen by the designer [12]. The matrices Q and R signify the trade-off between performance and control effort respectively. It should be noted that the control law that minimizes J is given by linear state-feedback u = −Kx.

Using MATLAB and the numerical values in Section **Error! Reference source not found.**, we find that,

$$K=\begin{bmatrix}1 & 0 & 0 & 1.649 & 0 & 0\\ 0 & 1 & 0 & 0 & 1.649 & 0\\ 0 & 0 & 1 & 0 & 0 & 1.649\end{bmatrix} \tag{17}$$

With a simple LQR, i.e, $\alpha=0$, all poles of state matrix A lie on an imaginary axis which makes the system becomes unstable. For solving this problem, we use the (Alpha) matrix to transfer poles from the imaginary axis. If we do so, a new controller matrix (K) can be achieved,

$$K=\left[857.3832I_{3\times3} \quad 254.7939I_{3\times3}\right] \tag{18}$$

With $\alpha=2$ and using control law u = −Kx, system will be stable as *Figure 3*.

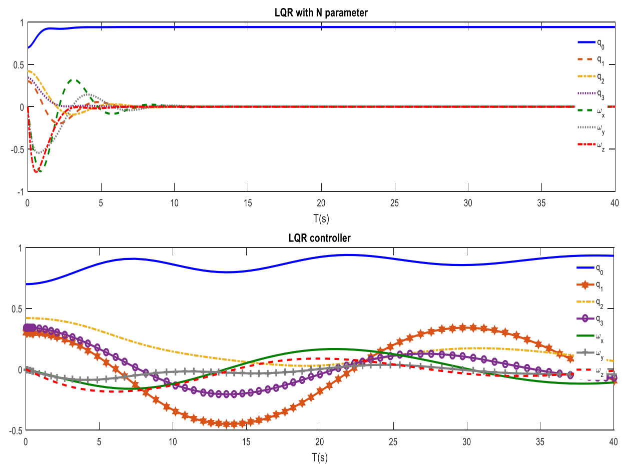

Figure 3 linear system with LQR controller and without controller

4.2. Angular system of Stewart platform

For the attitude control of the Stewart platform, the rotational movement can be extracted and simplified in order to design a feedback linearization controller. Feedback linearization transforms a non-linear system into a linear one, then by input-output feedback linearization the system is linearized and a state feedback control law is obtained by pole placement.

Now consider the angular position q and velocities ω of the platform represented by the system $\bar{S}_2$, It will rewrite

$$\begin{aligned}\dot{x}&=f(x)+g(x)u\\ y&=h(x)\end{aligned} \tag{20}$$

Where the quaternion vector is $\left[a \quad b \quad c \quad d\right]=\left[q_1 \quad q_2 \quad q_3 \quad q_4\right]$ and f(x) is nonlinear state matrix equals

$$f(x)=\begin{bmatrix}\frac{1}{2}(-\omega_x q_2-\omega_y q_3-\omega_z q_4)\\ \frac{1}{2}(\omega_x q_1+\omega_z q_3-\omega_y q_4)\\ \frac{1}{2}(\omega_y q_1-\omega_z q_2+\omega_x q_4)\\ a_1\omega_y\omega_z\\ a_2\omega_x\omega_z\\ a_3\omega_x\omega_y\end{bmatrix} \tag{21}$$

where the state variables are $\left[q_1 \quad q_2 \quad q_3 \quad q_4 \quad \omega_y \quad \omega_y \quad \omega_z\right]$ and

$$a_1=\frac{I_y-I_z}{I_x},\ a_2=\frac{I_z-I_x}{I_y},\ a_3=\frac{I_x-I_y}{I_z} \tag{22}$$

And $I_m=\left[I_x \quad I_y \quad I_z\right]$ is the top platform tensor of inertia. Input matrix in (18) is equal to

$$g(x)=\left[g_1(x) \quad g_2(x) \quad g_3(x)\right]=\begin{bmatrix} & \mathbf{0}_{4\times3} & \\ b_1 & 0 & 0\\ 0 & b_2 & 0\\ 0 & 0 & b_3\end{bmatrix} \tag{23}$$

Where $b_1=\frac{r_T\cos(120)}{I_x}, b_2=\frac{r_T}{I_y}, a_3=\frac{1}{I_z}$. Also output matrix of (20) is equal to $y=\left[h_1(x) \quad h_2(x) \quad h_3(x)\right]=\left[q_2 \quad q_3 \quad q_4\right]$.

4.3. FEEDBACK LINEARIZATION TECHNIQUE

This section deals with the design of a quaternion-based feedback control scheme for the purpose of transforming the nonlinear system (20) into a linear and controllable system. Each of the output components is differentiated a sufficient number of times until a control input component appears in the resulting equation. Using the Lie derivative, input-output linearization can transform the nonlinear system into a linear system. Then we can apply a linear control law for the linearized system.

4.4. INPUT-OUTPUT FEEDBACK LINEARIZATION TECHNIQUE

Furthermore, there are two feedback linearization methods that are [17-18]:

(a)- Input-state feedback linearization

(b)- Input-output feedback linearization

The input-output feedback linearization technique is summarized by three rules;

• Deriving output until input appears

• Choosing a new control variable which provides to reduce the tracking error and eliminate the nonlinearity

• Studying stability of the internal dynamics which are part of system dynamics cannot be observed in input-output linearization.

The vector relative degree of the system (20) is, $\begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T$ while the dimension of the system is 7. Since $r_1 + r_2 + r_3 = 6 < 7$, the nonlinear system can be input-output linearized only. Thus, we have [15]:

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ y_3^{(r_3)} \end{bmatrix} = D(x) + H(x) \tag{22}$$

Where $D(x)$ and $H(x)$ are computed as [13-14]:

$$D(x) = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \\ L_f^{r_3} h_3(x) \end{bmatrix}$$

$$H(x) = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \\ L_f^{r_3} h_3(x) \end{bmatrix} \times \begin{bmatrix} L_{g_1} L_f^{r_1 - 1} h_1(x) & \cdots & L_{g_3} L_f^{r_1 - 1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_3 - 1} h_3(x) & \cdots & L_{g_3} L_f^{r_3 - 1} h_3(x) \end{bmatrix} \tag{24}$$

Where the Lie derivatives are defined as [17-18]:

$$\begin{aligned} L_f h_i(x) &= \sum_{j=1}^{7} \frac{\partial h_i}{\partial x_i} f(x) \\ L_f^{r_i - 1} h_i(x) &= L_f (L_f^{r_i - 1} h_i(x)) = \sum_{j=1}^{7} \frac{\partial L_i^{r_i - 1} h_i}{\partial x_i} f(x) \\ L_{g_i} L_f^{r_i - 1} h_i(x) &= \sum_{j=1}^{7} \frac{\partial L_i^{r_i - 1} h_i}{\partial x_i} g_i(x), i = 1, 2, 3 \end{aligned} \tag{25}$$

The feedback linearization is feasible if and only if the matrix H(x) is nonsingular, which means that $|H(x)| \neq 0$. We can obtain the matrix H(x) by calculating Eq. (24)

$$\mathrm{H}(x) = \frac{1}{2} \begin{bmatrix} b_1 q_1 & -b_2 q_4 & b_3 q_3 \\ b_1 q_4 & b_2 q_1 & -b_3 q_2 \\ -b_1 q_3 & b_2 q_2 & b_3 q_1 \end{bmatrix} \tag{26}$$

And we will obtain:

$$|H(x)| = \frac{1}{8} b_1 b_2 b_3 (q_1^2 + q_2^2 + q_3^2 + q_4^2) q_1 = \frac{1}{8} b_1 b_2 b_3 q_1$$

when, $q_1 \neq 0$ the matrix H(x) is nonsingular and the input-output linearization problem is solvable for the nonlinear system of equation (18). By using the Feedback-Decoupling [15] of this multivariable system and Letting v = D(x) + H(x) u, we can compute the control law of the form [15, 19]:

$$u = H^{-1}(x)(v - D(x)) \tag{27}$$

4.5. Linear Control for Feedback Linearized System

Using the feedback linearization technique, the system (10) can be transformed into a system that, in suitable coordinates, is input-output linearized and controllable. The change of coordinates $\xi = \Phi(x)$ is given by

$$\begin{aligned} \xi_1 &= h_1(x) = q_2 & \xi_4 &= L_f h_1(x) = \dot{q}_2 \\ \xi_2 &= h_2(x) = q_3 & \xi_5 &= L_f h_1(x) = \dot{q}_3 \\ \xi_3 &= h_3(x) = q_4 & \xi_6 &= L_f h_1(x) = \dot{q}_4 \end{aligned} \tag{28}$$

In the new coordinates, the system appears as:

$$\begin{aligned} \dot{\xi} &= A\xi + Bv \\ y &= C\xi \end{aligned} \tag{29}$$

In which, $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 \end{bmatrix}$, and

$$A = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathrm{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} B = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathrm{I}_{3\times3} \end{bmatrix} C = \begin{bmatrix} \mathrm{I}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{30}$$

For the linear system (29), one can design a controller using a linear control law, which assigns the poles of the closed loop linear system to desired positions. The LQR controller law is used in this study. Once again we review the steps in section A to design controller law for linear system (29).

Using MATLAB and the numerical values in Section **Error! Reference source not found.**, we find that,

$$K = \begin{bmatrix} 400.4 I_{3\times3} & 40 I_{3\times3} \end{bmatrix} \tag{31}$$

In this controller law, we use the (Alpha) matrix to transfer poles from the imaginary axis. If we apply this controller law to the nonlinear system (20) behavior of the system states will be as follows:

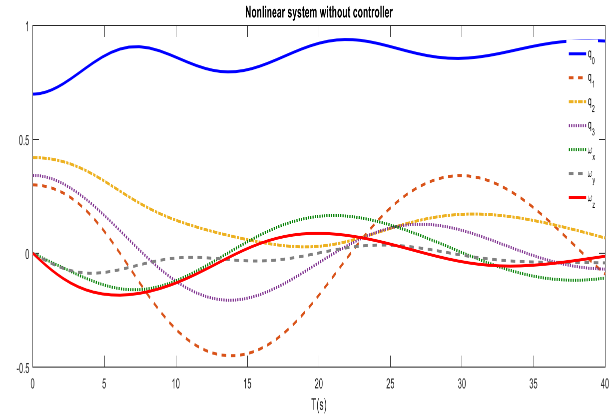

Figure 4 nonlinear system with controller K

As you can see the system was not stable ideally. We change the weighting coefficients matrix K for system stability. If we change the weighting coefficients matrix K of the nonlinear system (20) will be stable as

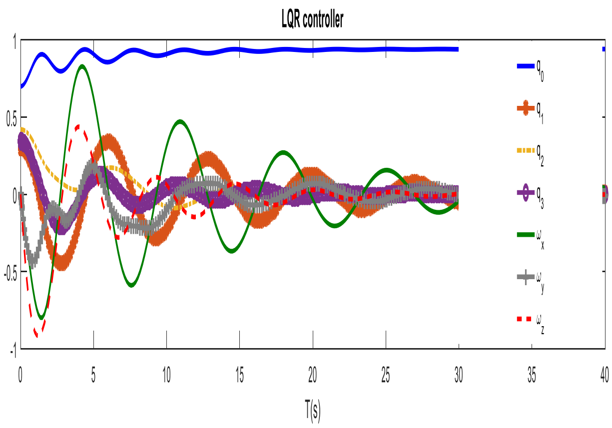

Figure 5 nonlinear system with new controller K and $\alpha = 10$

In this step, the controller matrix K will be as follows:

$$K = \begin{bmatrix} 5047.4 I_{3\times3} & 278.5605 I_{3\times3} \end{bmatrix} \quad (32)$$

Where the new weighting coefficients matrix in performance index J, will be as follows:

$$Q = \begin{bmatrix} \mathbf{0}_{3\times3} & 312500\mathrm{I}_{3\times3} \\ 312500\mathrm{I}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix}, R = 5\mathrm{I}_{3\times3} \quad (33)$$

5. Conclusions and future works

In this section, we analyze the results of the Stewart platform nonlinear system. As we saw in the previous section, the Stewart platform was stable. Now consider the state matrix A in (29), we're going to move the poles of this matrix and check the result on the stability of the Stewart platform. First, define the parameter of the Stewart platform in *Table 2*.

Table 2 Stewart platform parameter

Parameter	symbol	Initial value
Mass of platform (Kg)	***M***	***1.26***
Inrtial tansor (Kgm^2)	I_m	$10^{-4} \times diag[1.705 \quad 1.705 \quad 3.408]^T$
Gravity (m/s^2)	g	9.85
Sampling period (s)	T	0.01
Top platform radius (mm)	r_T	115
Initial position,Top platform	P_0	[0,0,150]
Initial orientation, Top platform	q_0	[0.698,0.3,0.42,0.342]

Using the matrix alpha to change the pole position of state matrix A and then check the stability of the platform. So, we change alpha from 1 to 100 and see the effect on the change of maximum weighting coefficient in the K matrix.

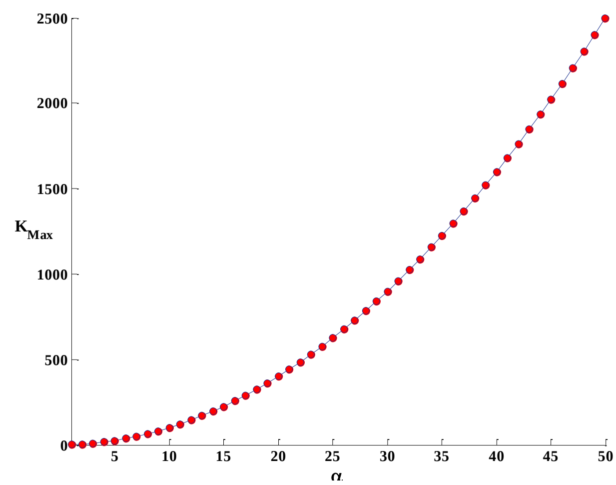

Figure 6 maximum weighting coefficient of K with respect to α

With this change to the state matrix of platform, we'll review how to change the controller parameters and the Stewart platform. *Figure 7* and *Figure 8* show the change of maximum and minimum overshoot when we apply the LQR controller on the Stewart platform.

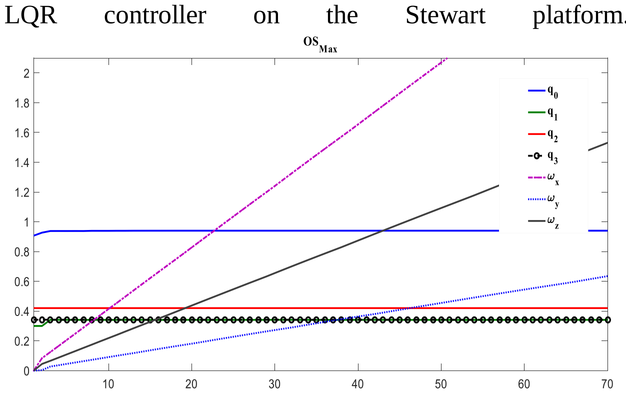

Figure 7 maximum overshoot of platform state with first K

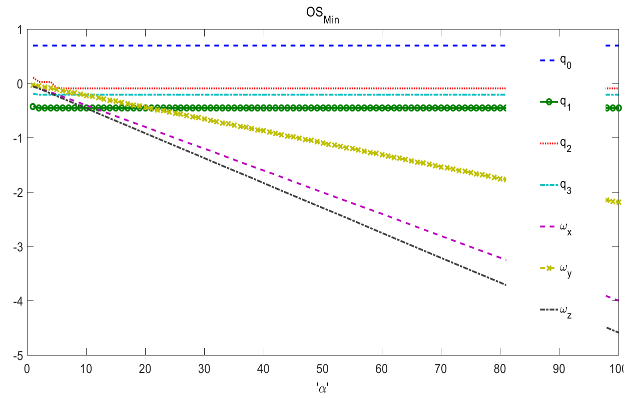

Figure 8 minimum overshoot of platform state with first K

It is obvious that by increasing the amount of alpha, the overshoots also increase. If we apply this change on the corrected coefficient matrix LQR controller (K), this change will be as *Figure 9* and *Figure 10*.

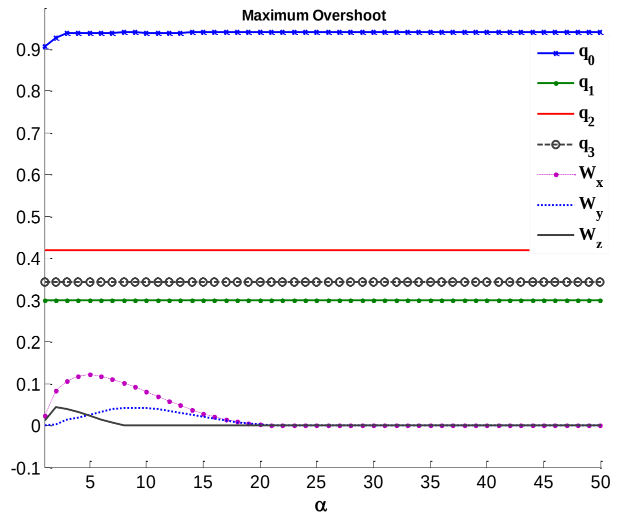

Figure 9 maximum overshoot of platform state with corrected coefficient K

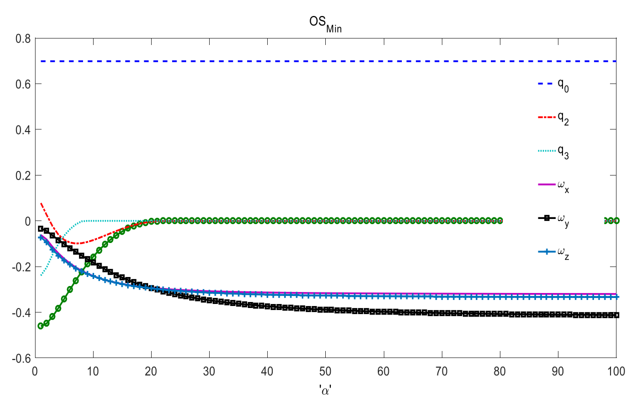

Figure 10 minimum overshoot of platform state with corrected coefficient K

It can be seen that by increasing the alpha value of 70, the lowest overshoot achieved. *Figure 11* show the nonlinear system with effects of alpha=70 on the LQR controller matrix (K).

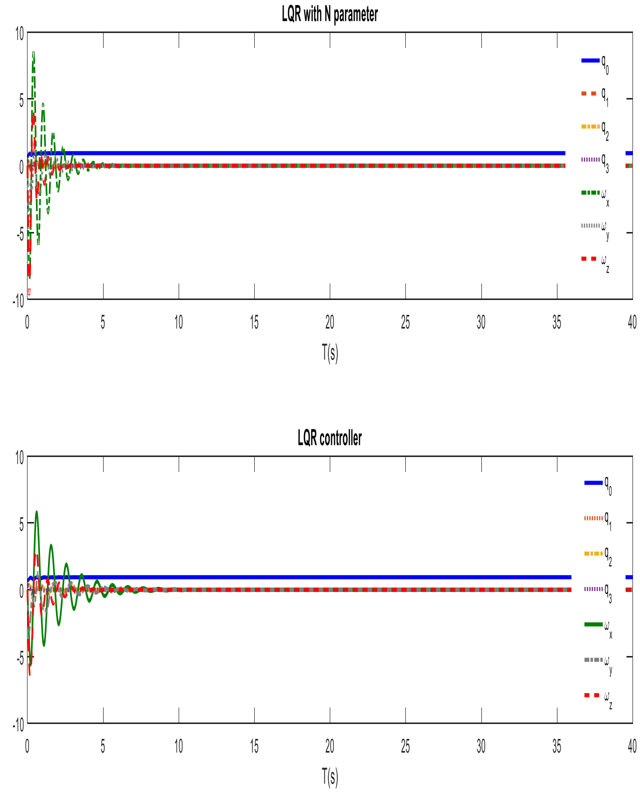

Figure 11 nonlinear system with corrected coefficient K controller and alpha=70

For future work, someone can use model predictive control (MPC) for feedback-linearized Stewart platform or using neural-network model predictive control (NNMPC) which it may to better results to LQR for feedback-linearized quaternion model.

References

[1] D.stewart, A Platform with Six Degrees of Freedom," *Proc.Inst. Mech. Engr.*, (1965), 371–386

[2] K. Liu, G. Lebret, and F. L. Lewis, "Dynamic analysis and control of a stewart platform manipulator," *Journal of Robotic systems*, vol. 10, no. 5, (1993), 629-655.

[3] M. Ayas, E. Sahin, and I. Altas, , "Trajectory tracking control of a stewart platform," Power Electronics and Motion Control Conference and Exposition (PEMC), vol. 16, (2014), 720–724.

[4] P. Kumar, A. Chalanga, and B. Bandyopadhyay, "Position control of stewart platform using continuous higher order sliding mode control," Asian Control Conference (ASCC), vol. 10, (2015),1-6.

[5] S. Maged, A. Abouelsoud, and A. Fath El Bab, "A comparative study of unscented and extended kalman filter for position and velocity estimation of stewart platform manipulator," *Cyber Technology in Automation, Control, and Intelligent Systems (CYBER), 2015 IEEE International Conference*, (2015), 1216-1222.

[6] K. H. Hunt, Kinematic geometry of mechanisms, Oxford University Press, (1978).

[7] H. Gonzalez, M. S. Dutra, and O. Lengerke, "Direct and inverse kinematics of stewart platform applied to offshore cargo transfer simulation," *13th World Congress in Mechanism and Machine Science,* (2011).

[8] Maleki Roudposhti M , Agheli Hajiabadi M , Design, Fabrication, and Kinematic Analysis of a 6 DOF Mobile Wheeled Parallel Robot, Modares Mechanical Engineering, Vol. 20, No. 7, (2020), 1749-1759.

[9] J. Sharifi and M. Doustani, Sensitivity Analysis of a New Model of Stewart Platform, Mathematics and Computational Sciences, Vol. 3, No. 3, (2022), 47-56.

[10] Tamir, Tariku Sinshaw, et al. "Design and optimization of a control framework for robot assisted additive manufacturing based on the stewart platform," International Journal of Control, Automation and Systems, Vol. 20, No. 3, (2022), 968-982

[11] Vu, Mai The, et al. "Optimized Fuzzy Enhanced Robust Control Design for a Stewart Parallel Robot." Mathematics, Vol. 10, No. 11, (2022).

[12] Du, Shang Jian, “Simulation and tool path optimization for the hexapod milling machine”, Vol. 26. Vulkan-Verlag GmbH, (2005).

[13] Paulo F. S. R. de Faria, Rafael S. Castro, Aur´elio T. Salton and Jeferson V. Flores, "Quaternion-based Dynamic Control of a 6-DOF Stewart," in 2016 IEEE Conference on Control Applications (CCA), Buenos Aires, Argentina, 2016.

[14] Ali Khaki-Sedigh, “Modern Control Principles ”, 12th edition, , University of Tehran Publisher, 2014.

[15] P. Albertos and A. Sala, “Multivariable Control

Systems: An Engineering Approach”, Springer-Verlag. 2004.

[16] G. F. Franklin, J. D. Powell, A. Emami-Naeini , Feedback Control of Dynamic Systems, 5th ed. , Upper Saddle River: Pearson Prentice Hall, 2006.

[17] H. Nijmeijer and A. van der Schaft. Nonlinear Dynamical Control Systems, Springer-Verlag, 1990.

[18] Isidori, Alberto, Nonlinear control systems: an introduction. Berlin, Heidelberg: Springer Berlin Heidelberg, 1985.

[19] Ali Khaki-Sedigh, “Analysis and Design of Multivariable Control Systems”, K. N. Toosi University of Technology, 2020.

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