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A New Framework of Synchronized Adaptive Fuzzy Sliding Mode Control for Networked Under-Actuated Systems Subjected to Communication Delay

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ABSTRACT

A new framework of synchronized adaptive fuzzy sliding mode control (AFSMC) approach for a network of under-actuated systems (UASs) under communication time delay is presented here. The basic equations of motion of each agent for controller design and information exchange paradigm among them are considered as cascaded normal form and master/slave, respectively. A fuzzy system is applied to determine the equivalent part of the controller which is based on classical sliding mode control (SMC). Then, its robust part is improved in comparison with the conventional AFSMC so as to synchronize the agents to the leader's state. In addition to synchronization, the proposed AFSMC improves some properties associated with the transient part of the response, especially rise-time, significantly. The proposed scheme is robust against uncertainties and unknown communication time delay, as well. Also, its implementation is so simple that different UASs can be replaced, easily. Moreover, the presented controller is completely model-free for UASs with strict feedback form dynamics and less-dependent on the dynamics of UASs with cascaded normal form.

1. Introduction

Under-actuated systems (UASs) are a kind of mechanical systems which have fewer control input in comparison with the system's configuration variables. This makes their control a challenging problem. However, they have received considerable attentions due to their wide applications in aerospace (satellites, vertical take-off and landing (VTOL) aircrafts, multi-rotor unmanned aerial vehicles (UAVs), helicopters), robotics (robots with flexible links, mobile robots, snake-type, and swimming robots), surface vessels, and underwater vehicles [2]. On the other hand, when they come to collaboration in collective motions other issues such as communication time delay is added to the underactuation problem.

Despite the aforementioned difficulty in control of UASs, extensive efforts have been made by researchers to overcome their control challenge. Of most important works in this field, [3] and [4] can be noted. In [3], a method is proposed by which the equations of motion are partially linearized. However, the control input appears in both equations corresponding to actuated and unactuated configuration variables. Olfati-Saber in [4], presented a classification of UASs in general form. He also presented a systematic approach by which the equations of motion of UASs can be written as a cascaded normal form. The appearance problem of

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control input in both actuated and unactuated related equations vanishes by this form. In addition, dealing with cascaded normal forms facilitate control synthesis, considerably.

On the other hand, synchronization of the systems especially chaotic or more generally fractional-order chaotic systems have been widely studied, recently [5-7]. In [5], Davy and Jiang presented an adaptive back-stepping controller based on Lyapunov theory to achieve chaos synchronization of two gyros in which both of them contain chaotic and nonlinear behavior and have some uncertainties. In [6], Sachin and Varsha addressed an active control technique to synchronize various fractional order chaotic dynamical systems. In [7], Ahmad-Taher mentioned synchronization of similar chaotic systems using sliding mode control in hybrid phase. Synchronization control is employed in many applications such as: cooperative robot manipulators [8], autonomous underwater vehicle [9], motion tracking control of numerous underactuated ships [10], formation control of under-actuated autonomous surface vehicles [11]. Some other recent studies on synchronization of multi agent systems (MASs) can be found in [12-14].

The AFSMC approach which is synthesized by integration of SMC method and fuzzy logic based controllers (FLC), has attracted a lot of interests during the last two decades. Each of these two methods has its own unique pros and cons. The SMC is extensively used for robust control of nonlinear systems which have uncertainties and external disturbances. However, it has some disadvantages; like the chattering problem which leads to undesirable behavior in steady-state part of system's response. On the other hand, the FLC could be used for control of the systems for which there is not complete information about the process or dynamic model. Lack of a systematic approach to design a stabilizing controller and difficulty in adjusting parameters are two challenges related to FLC. In order to take advantage of the benefits of both techniques and overcome their problems, some hybrid methods such as AFSMC are proposed and widely studied [15-22].

The AFSMC has also been applied to synchronization of chaotic systems, e.g. [23] and [24]. In [23] the AFSMC scheme is established in order to synchronize fractional-order chaotic systems. In [24] the method is extended for uncertain fractional-order chaotic systems qwwith intrinsic time delay. Recently, another branch of AFSMC approach which utilizes type-2 fuzzy membership functions (MFs) has been used for better handling the uncertainties [25, 26]. Mohammadzadeh et al. established self-structuring hierarchical to synchronize fractional order chaotic systems [25], and self-evolving nonsingleton type-2 fuzzy neural networks [26]. Several other papers can be cited in this regard [27-29].

The cooperation paradigm in the published literature on synchronization of uncertain systems by AFSMC is considered as the master/slave with one agent as the master and one agent as the slave. However, the communication topology has not been included in the controller design procedure. In other words, the AFSMC method has not been extended for synchronization of multi-agent (more than 2 agents) systems with master/slave paradigm so far. Furthermore, the communication time delay in transmitting output signal among the agents has been neglected in [23-29]. Hence, the main contribution of this paper is the improvement of the AFSMC method for synchronization of networked multi-agent UASs subjected to communication delay. The equivalent part of the controller is estimated by using universal approximation capability of fuzzy systems and the robust part is modified such that the proposed controller has the following features:

1. The communication topology and time delay has been considered during control synthesis.
2. The synthesized controller improves some properties associated with the transient part of the response, especially rise-time, in comparison with the conventional AFSMC.

The organization of the subsequent sections in this paper is as follows: Section 2 summarizes the equations of motion of UASs in the original and cascaded normal forms. In section 3 the proposed AFSMC method for synchronization of networked UASs under communication time delay and leader/follower cooperation paradigm is synthesized and followed by a theorem. The results of implementation of the proposed controller on two UASs, i.e. rotating inverted pendulum and quadrotor are presented in section 4. Section 5 concludes the paper.

2. Under-actuated Systems Dynamics

Consider the following equations of motion for a class of UASs with two configuration variables and one independent control input:

$$\begin{aligned} m_{11}(q)\ddot{q}_1 + m_{12}(q)\ddot{q}_2 + h_1(q,\dot{q}) &= 0 \\ m_{21}(q)\ddot{q}_1 + m_{22}(q)\ddot{q}_2 + h_2(q,\dot{q}) &= \tau \end{aligned} \tag{1}$$

Here $q = [q_1, q_2], \dot{q}$ are the vector of system states, τ is the control input and h_i includes Coriolis, centrifugal force, and gravity related terms. A systematic approach for transformation of the original equations into a cascaded normal form is presented in [4]:

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1 \\ \dot{x}_2 &= f_1(x_1, x_2, x_3, x_4) + d_2 \\ \dot{x}_3 &= x_4 \\ \dot{x} &= f_2(x_1, x_2, x_3, x_4) + g(x_1, x_2, x_3, x_4)u + d_3 \end{aligned} \tag{2}$$

Where $x_i \in R^n (i = 1,2,3,4)$ is the system's state variable in the cascaded normal form, $u \in R^n$ is the control input, $f_1, f_2 : R^{4n} \to R^n, g : R^{4n} \to R^{n\times n}$ are linear smooth vector functions. Furthermore g is invertible and $d_i \in R^n (i = 1,2,3)$ represents the disturbances. The equations of motion of most UASs can be rewritten in the cascaded forms. Inverted pendulum system, VTOL aircrafts, and quadrotors are examples of such UASs.

We rewrite the equations of motion for a network of N under-actuated agents, locally:

$$\begin{aligned} \dot{x}_i^1 &= x_i^2 \\ \dot{x}_i^2 &= f_i^1 + d_i^1 \\ \dot{x}_i^3 &= x_i^4 \\ \dot{x}_i^4 &= f_i^2 + g_i^1 u_i + d_i^2 \end{aligned} \tag{3}$$

where $i = 1, 2, \ldots, N$ In the global form we have:

$$\begin{aligned} \dot{x}^1 &= x^2 \\ \dot{x}^2 &= f^1 + d^1 \\ \dot{x}^3 &= x^4 \\ \dot{x}^4 &= f^2 + g^1 u + d^2 \end{aligned} \tag{4}$$

where $x^j = [x_1^j, x_2^j, \ldots, x_N^j], j = 1,2,3,4$, $f^k = [f_1^k, f_2^k, \ldots, f_N^i]^T, k = 1,2$, $u = [u_1, u_2, \ldots, u_N]^T$, and $g^1 = diag(g_1^1, g_2^1, \ldots, g_N^1)$. In this paper, the paradigm for exchange of information among agents in the networked system is considered as leader/follower or master/slave. The communication topology is also considered as fixed and its associated graph, G, as a weakly connected digraph with spanning tree. The communication links is also subjected to unknown communication time delay.

Definition 1. The local neighbourhood synchronization errors up to 4th order for i th UAS are defined as follows:

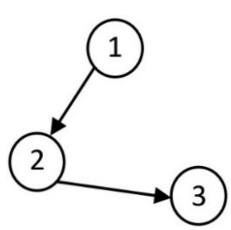


Fig. 1. Communication topology.

$$\begin{aligned} e_i^1 &= \sum_{j\in N_i} a_{ij}\left(x_j^1(t-T) - x_i^1(t)\right) \\ &\quad + \sum_{j\in N_i} b_i\left(x_0^1(t-T) - x_i^1(t)\right) \\ e_i^2 &= \sum_{j\in N_i} a_{ij}\left(x_j^2(t-T) - x_i^2(t)\right) \\ &\quad + \sum_{j\in N_i} b_i\left(x_0^2(t-T) - x_i^2(t)\right) \\ e_i^3 &= \sum_{j\in N_i} a_{ij}\left(f_j^1(t-T) - f_i^1(t)\right) \\ &\quad + \sum_{j\in N_i} b_i\left(\ddot{x}_0^1(t-T) - f_i^1(t)\right) \\ e_i^4 &= \sum_{j\in N_i} a_{ij}\left(Jf_j^1(t-T) - Jf_i^1(t)\right) \\ &\quad + \sum_{j\in N_i} b_i\left(\dddot{x}_0^1(t-T) - Jf_i^1(t)\right) \end{aligned} \tag{5}$$

where $x_0(t)$ is the state of the leader or control node which is also known as consensus value, $b_i \ge 0$ is pinning gain and at least for one agent is nonzero which indicates that that agent receives direct information from the leader. T is unknown communication time delay. $A = \left[a_{ij}\right]$ is adjacency matrix which its elements denote the edge weight in communication graph. In addition, Jf_i^1 is the Jacobin of f_i^1 and is obtained as:

$$Jf_i^1 = \frac{\partial f_i^1}{\partial x_i^1} x_i^2 + \frac{\partial f_i^1}{\partial x_i^2} f_i^1 + \frac{\partial f_i^1}{\partial x_i^3} x_i^4 \tag{6}$$

Therefore, the synchronization problem is defined as: Design the distributed control protocols for all nodes such that the followers synchronize to the state of the leader, i.e. $\forall_i, x_i(t) \to x_0(t)$. It is assumed that the dynamics of the leader for all nodes in G is unknown. It is further assumed that the dynamics of each agent for its corresponding controller is unknown or at least a rough estimation of some parameters exist.

3. Synchronization Control Strategy

Robust control methods should be used to make significant progress in dealing with the uncertainty of the system. In this study, the AFSMC approach is opted to control the system. As mentioned above, the infrastructure of AFSMC is SMC method where fuzzy and adaptive laws are added to deal with unknown uncertainties. Therefore, the sliding surface is defined first [30]:

$$s_i = c_{i1} e_i^1 + c_{i2} e_i^2 + c_{i3} e_i^3 + e_i^4 \tag{7}$$

According to Eq. (5), it can be concluded that $\dot{e}_i^1 = e_i^2, \dot{e}_i^2 = e_i^3$ and $\dot{e}_i^3 = e_i^4$. Time derivative of Eq. (7) yields:

$$\dot{s}_i = c_{i1} e_i^2 + c_{i2} e_i^3 + c_{i3} e_i^4 + \dot{e}_i^4 \tag{8}$$

Also, with respect to Eq. (6), one can write:

$$\frac{d}{dt}Jf_i^1=\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^1}x_i^2+\frac{\partial f_i^1}{\partial x_i^2}f_i^2\right)+\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^3}\right)x_i^4+\frac{\partial f_i^1}{\partial x_i^3}\dot{x}_i^4 \tag{9}$$

As a result, $\dot{s}$ equals to:

$$\begin{aligned}\dot{s}_i=&c_{i1}e_i^2+c_{i2}e_i^3+c_{i3}e_i^4+\sum_{j\in N_i}a_{ij}\frac{d}{dt}\left(Jf_j^1(t)\right)\\&+b_i\frac{d}{dt}\left(\ddot{x}_0^1(t)\right)\\&-\left(\sum_{j\in N_i}a_{ij}+b_i\right)\times\left\{\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^1}x_i^2++\frac{\partial f_i^1}{\partial x_i^2}f_1^1\right)\right.\\&\left.+\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^3}\right)x_i^4+\frac{\partial f_i^1}{\partial x_i^3}\left(f_i^2+g_i^1u_i+d_i^2\right)\right\}\end{aligned} \tag{10}$$

To extract the equivalent part of the controller, u_i^{eq}, $\dot{s}$ is set to zero:

$$\begin{aligned}u_i^{eq}=&\frac{1}{\left(\sum_{j\in N_i}a_{ij}+b_i\right)\frac{\partial f_i^1}{\partial x_i^3}g_i^1}\left\{c_{i1}e_i^2+c_{i2}e_i^3+c_{i3}e_i^4\right.\\&+\sum_{j=N_i}a_{ij}\frac{d}{dt}\left(Jf_j^1(t)\right)+b_i\frac{d}{dt}\left(\ddot{x}_0^1(t)\right)\\&-\left(\sum_{j\in N_i}a_{ij}+b_i\right)\left[\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^1}x_i^2+\frac{\partial f_i^1}{\partial x_i^2}f_i^1\right)\right.\\&\left.\left.+\frac{d}{dt}\left(\frac{\partial f_i^1}{\partial x_i^3}\right)x_i^4+\frac{\partial f_i^1}{\partial x_i^3}f_i^2\right]\right\}\end{aligned} \tag{11}$$

From Eqs. (10) and (11), $\dot{s}$ can be rewritten:

$$\begin{aligned}\dot{s}_i=&\left(\sum_{j\in N_i}a_{ij}+b_i\right)\frac{\partial f_i^1}{\partial x_i^3}g_i^1\\&\times\left\{u_i^{eq}-u_i-\left(g_i^1\right)^{-1}d_i^2\right\}\end{aligned} \tag{12}$$

As can be seen from Eq. (11), u_i^{eq} is complex and depends on the system dynamics, completely. As mentioned, since the agents are going to be unaware of their dynamics and the dynamics of the leader, the control input should be independent of the nodes dynamics, as much as possible. Therefore, The approximation theorem of fuzzy systems is utilized for approximating u_i^{eq}. This approximation is indicated by $\hat{u}_{i,fuzz}$. For this purpose, we use the fuzzy system of Takagi-Sugeno-Kung (TSK) type with input s and output. $\hat{u}_{i,fuzz}$. Therefore, the if-then rules of fuzzy systems are expressed as:

Rule r: if s_i is A_i^r then $\hat{u}_{i,fuzz}=\theta_i^r$ for $r=1,\dots,n_r$

Here, A_i^r is MF of fuzzy system, which is defined as a Gaussian function:

$$\mu_{A_i^r}(s_i)=\exp\left[-\left(\frac{s_i-c_i^r}{\sigma_i^r}\right)^2\right] \tag{13}$$

θ_i^r is a fuzzy singleton that is related to output of i th rule, c_i and σ_i are the centre and variance of MFs, respectively. Using the singleton fuzzifier, product inference engine, and centre-average defuzzifier, the output of TSK system for i th agent is:

$$\hat{u}_{i,fuzz}(s_i)=\frac{\sum_{r=1}^{n_r}\theta_i^r\mu_{A_i^r}(s_i)}{\sum_{r=1}^{n_r}\mu_{A_i^r}(s_i)} \tag{14}$$

The above relation can be written as follows:

$$u_{i,fuzz}\left(s_i\left|\theta_i\right.\right)=\theta_i^TW_i \tag{15}$$

where:

$$\begin{aligned}&\theta_i=\left[\theta_i^1,\dots,\theta_i^{n_r}\right]^T,W_i=\left[W_i^1,\dots,W_i^{n_r}\right]^T\\&W_i^r=\frac{\mu_{A_i^r}(s_i)}{\sum_{r=1}^{n_r}\mu_{A_i^r}(s_i)}\end{aligned} \tag{16}$$

This approximation has an optimal value as $u_{i,fuzz}^*$:

$$u_i^{eq}=u_{i,fuzz}^*+\Delta_i \tag{17}$$

Here, Δ_i is the minimum approximation error. Therefore, the applied u_i is considered as:

$$u_i=\hat{u}_{i,fuzz}+u_i^{rb} \tag{18}$$

Where $\hat{u}_{i,fuzz}\left(s_i\left|\hat{\theta}_i\right.\right)=\hat{\theta}_i^TW_i$ and $\hat{\theta}_i$ is the approximation of θ_i. Moreover u_i^{rb} is the robust part of the control input. The difference between estimated value of u_i^{eq} and the optimal approximation value is shown with $\tilde{u}_{i,fuzz}$:

$$\begin{aligned}\tilde{u}_{i,fuzz}&=u_{i,fuzz}^*-\hat{u}_{i,fuzz}\\&=\left(\theta^*\right)^TW-\left(\hat{\theta}\right)^TW\\&=\left(\theta^*-\hat{\theta}\right)^TW\end{aligned} \tag{19}$$

So:

$$\begin{aligned}\dot{s}_i=&\left(\sum_{j\in N_i}a_{ij}+b_i\right)\frac{\partial f_i^1}{\partial x_i^3}\\&\times g_i^1\left\{u_{i,fuzz}^*+\Delta_i-\hat{u}_{i,fuzz}-u_i^{rb}\right\}\end{aligned} \tag{20}$$

The robust part of controller, u_i^{rb} for i th follower agent is proposed as:

$$u_i^{f-rb} = \left[\frac{\partial f_i^1}{\partial x_i^3} g_i^1\right]^{-1} \left\{ \frac{c}{\sum\limits_{j\in N_i} a_{ij}+b_i} s_i + k_i - \frac{1}{\sum\limits_{j\in N_i} a_{ij}+b_i}\left(s_j(t-T)-s_i(t)\right) \right\} \tag{21}$$

and for leader as:

$$u_i^{l-rb} = \left[\frac{\partial f_i^1}{\partial x_i^3} g_i^1\right]^{-1} \left(cs_i + k_i\right) \tag{22}$$

It can be figured out from Eq. (21) that the controller's dependence on the dynamics is $\left[\frac{\partial f_i^1}{\partial x_i^3} g_i^1\right]^{-1}$. On the other hand, as explained in [4], some of UASs such as Acrobat, inertia-wheel pendulum, TORA system, and rotating inverted pendulum belong to class-I of UASs which their equations of motion can be rewritten in the strict feedback form. In the strict feedback from, the term $\left[\frac{\partial f_i^1}{\partial x_i^3} g_i^1\right]^{-1}$ is equal to 1 and hence, the controller is independent of the dynamics, completely, whereas for other UASs such as quadrotor that cannot be simpler than the cascaded normal form, the controller has a little dependency on the system dynamics. With these explanations, now the first result in the synchronization of networked UASs is presented under communication delay and weakly connected spanning tree graph topology.

Theorem 1. Distributed AFSMC protocols for synchronization of networked under-actuated agents under communication delay: Consider a network of UASs, which their equations are given by Eqs. (3) and (4). If the agents in the network exchange s_i signals with an unknown time delay T over a weakly-connected communication graph with spanning tree, with the controllers given in Eqs. (15), (21) and (22), as well as the adaptive rules presented in Eq. (23) for followers and Eq. (24) for the leader, the agents achieve synchronous motion and asymptotically converge to consensus value.

$$\begin{aligned} \dot{\hat\theta}_i^f &= -\Gamma_i W_i s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i\right)\frac{\partial f_i^1}{\partial x_i^3} g_i^1 \\ \dot{\hat\xi}_i^f &= -F_i \psi_i s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i\right) \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{\hat\theta}_i^l &= -\Gamma_i W_i s_i^T \frac{\partial f_i^1}{\partial x_i^3} g_i^1 \\ \dot{\hat\xi}_i^l &= -F_i \psi_i s_i^T \end{aligned} \tag{24}$$

Proof. We first examine the stability of the followers with the given controller. Also, we will remove the superiors of the leader and the follower except where it leads to confusion. Let define $\phi_i \in [t-T,t], z_{it} = z_i(\phi_i), z_i = \left[s_i, \tilde\theta_i, \tilde\xi_i\right]^T$ and, $Z_t = \left[z_{1t}^T, ..., z_{Nt}^T\right]$ The following Lyapunov function is selected:

$$V(Z_t) = \frac{1}{2}\sum_{i=1}^N \left(s_i^T s_i + \tilde\theta_i \Gamma_i^{-1}\tilde\theta_i + \tilde\xi_i^T F_i^{-1}\tilde\xi_i + \sum_{j\in N_i} a_{ij} \int_{t-T}^{t} s_i^T(\sigma) s_i(\sigma) d\sigma \right) \tag{25}$$

Differentiating the above equations with respect to time and substituting Eqs. (15), (20), and (21):

$$\begin{aligned} \dot V(Z_t) = \sum_{i=1}^N \Bigg\{ & s_i^T \Bigg[\left(\sum_{j\in N_i} a_{ij} + b_i\right)\frac{\partial f_i^1}{\partial x_i^3} g_i^1\left(\tilde\theta_i^T W_i + \Delta_i\right) - c_i s_i \\ & - \left(\sum_{j\in N_i} a_{ij} + b_i\right) k_i + \sum_{j\in N_i} a_{ij}\left(s_j(t-T) - s_i(t)\right)\Bigg] \\ & + \tilde\theta_i^T \Gamma_i^{-1}\dot{\tilde\theta}_i + \tilde\xi_i^T F_i^{-1}\dot{\tilde\xi}_i \\ & + \sum_{j\in N_i} \frac{1}{2} a_{ij}\left(s_i^T(t)s_i(t) - s_i^T(t-T)s_i(t-T)\right)\Bigg\} \end{aligned} \tag{26}$$

By performing algebraic operations on Eq. (26), the result is:

$$\begin{aligned} \dot V(Z_t) = & -\sum_{i=1}^N c_i s_i^T s_i + \sum_{i=1}^N s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i\right)\left[\frac{\partial f_i^1}{\partial x_i^3} g_i^1 \Delta_i - k_i\right] \\ & + \sum_{i=1}^N \tilde\theta_i^T \left(\Gamma_i^{-1}\dot{\tilde\theta}_i + W_i s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i\right)\frac{\partial f_i^1}{\partial x_i^3} g_i^1\right) \\ & + \sum_{i=1}^N \tilde\xi_i^T F_i^{-1}\dot{\tilde\xi}_i + \sum_{i=1}^N \sum_{j\in N_i} a_{ij} s_i^T(t)\left(s_j(t-T) - s_i(t)\right) \\ & + \sum_{i=1}^N \sum_{j\in N_i} \frac{1}{2} a_{ij}\left(s_i^T(t)s_i(t) - s_i^T(t-T)s_i(t-T)\right) \end{aligned} \tag{27}$$

The k_i function is used to approximate the part $\eta_i sign(s_i)$ of the controller and to eliminate the chattering phenomenon. We approximate this function with the fuzzy system:

$$\begin{aligned} & k_i = \hat\xi_i^T \psi_i \ , \ \tilde\xi_i = \xi_i^* - \hat\xi_i \\ & \Rightarrow \hat\xi_i = \xi_i^* - \tilde\xi_i \\ & \Rightarrow \hat\xi_i^T \psi_i = \left(\xi_i^* - \tilde\xi_i\right)^T \psi_i \\ & k_i = \left(\xi_i^* - \tilde\xi_i\right)^T \psi_i \end{aligned} \tag{28}$$

In the above relations, the * indicates the optimal value of the estimate. By doing some algebraic operations, substituting k_i and the adaptive rules Eq. (23) and Eq. (27):

$$\dot{V}(Z_t) = -\sum_{i=1}^{N} c_i s_i^T s_i + \sum_{i=1}^{N} s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i \right) \times \left\{ \frac{\partial f_i^1}{\partial x_i^3} g_i^1 \Delta_i - \left(\xi_i^*\right)^T \psi \right\} + \sum_{i=1}^{N} \sum_{j\in N_i} a_{ij} s_i^T(t)\left(s_j(t-T) - s_i(t)\right) + \sum_{i=1}^{N} \sum_{j\in N_i} \frac{1}{2} a_{ij} \left(s_i^T(t) s_i(t) - s_i^T(t-T) s_i(t-T)\right) \tag{29}$$

Since each node does not have any delay in the exchange of information with itself, so the above relation can be rewritten as follows:

$$\dot{V}(Z_t) = -\sum_{i=1}^{N} c_i s_i^T s_i + \sum_{i=1}^{N} s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i \right) \left\{ \frac{\partial f_i^1}{\partial x_i^3} g_i^1 \Delta_i - \left(\xi_i^*\right)^T \psi \right\} + \sum_{i=1}^{N} \sum_{j\in N_i} a_{ij} s_i^T(t)\left(s_j(t-T) - s_i(t)\right) + \sum_{i=1}^{N} \sum_{j\in N_i} \frac{1}{2} a_{ij} \left(s_i^T(t) s_i(t) - s_j^T(t-T) s_j(t-T)\right) \tag{30}$$

Simplification of the above relation gives:

$$\dot{V}(Z_t) = -\sum_{i=1}^{N} c_i s_i^T s_i + \sum_{i=1}^{N} s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i \right) \times \left\{ \frac{\partial f_i^1}{\partial x_i^3} g_i^1 \Delta_i - \left(\xi_i^*\right)^T \psi \right\} - \frac{1}{2} \sum_{i=1}^{N} \sum_{j\in N_i} a_{ij} \left(s_i(t) - s_j(t-T)\right)^2 \tag{31}$$

The minimum error approximation is defined as follows:

$$w_i = \frac{\partial f_i^1}{\partial x_i^3} g_i^1 \Delta_i - \left(\xi_i^*\right)^T \psi \tag{32}$$

In this case $\dot{V}$ changes as follows:

$$\dot{V}(Z_t) = \sum_{i=1}^{N} \left[-c s_i^T s_i + s_i^T \left(\sum_{j\in N_i} a_{ij} + b_i \right) w_i - \frac{1}{2} \sum_{j\in N_i} a_{ij} \left(s_i(t) - s_j(t-T)\right)^2 \right] \tag{33}$$

According to the universal theorem of approximation for fuzzy systems, w_i can be reduced enough such that the following inequality holds:

$$\dot{V}(Z_t) \le -\sum_{i=1}^{N} \left[c\left\|s_i\right\|^2 + \sum_{j\in N_i} a_{ij} \left\|s_i(t) - s_j(t-T)\right\|^2 \right] \tag{34}$$

This inequality and the definition of Lyapunov candidate function in Eq. (25) confirm the boundedness of $s_i, \hat{\theta}_i, s_i(t) - s_i(t-T), s_i(t) - s_j(t-T)$. Based on the discussion in [31] and [32], for every agent in the group the values of $s_i(t), s_i(t) - s_i(t-T), s_i(t) - s_j(t-T)$ converge to zero, asymptotically. Furthermore, considering the sliding surface definition, it can be concluded that for each agent, which is in direct communication with the leader, the values of $x_i'(t) - x_0'(t)$ and for other agents the values of $x_i'(t) - x_j'(t)$ converge to zero asymptotically. As a result, the agents achieve synchronization in motion and follow the reference state of the leader. The same proof procedure can be carried out for the leader agent with this exception that the term $\frac{1}{2}\sum_{i=1}^{N} \left(\sum_{j\in N_i} a_{ij} \int_{t-T}^{t} s_i^T(\sigma) s_i(\sigma) d\sigma \right)$ is removed from the Lyapunov candidate function.

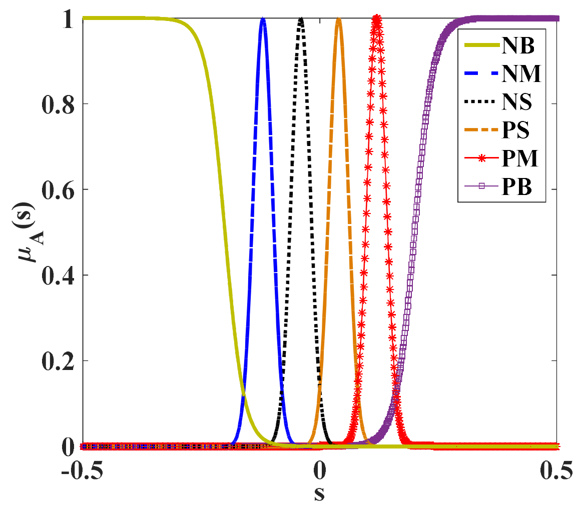


Fig. 2. The set of MFs for sliding surface fuzzification

4. Simulation

In this section, numerical results of implementation of the proposed controller on two UASs are presented. These systems have motion equations similar to Eq. (2). The communication graph for both examples are similar and are as shown in Fig. 1. The agent 1 is leader and agents 2 and 3 are followers. Throughout this paper, the set of MFs are considered as Fig. 2, in which NB, NM, NS, PS, PM, and PB stand for negative big, negative medium, negative small, positive small, positive medium, and positive big, respectively.

4.1. Rotating Inverted Pendulum

The rotating inverted pendulum (RIP) is a system that includes an inverted pendulum attached to a rotating arm located in a horizontal plane, as indicated in Fig. 3. The
motion equations of this system can be written for i th agent as follows [1]:
motion equations of this system can be written for i th agent as follows [1]:

$$\begin{aligned}
\dot{q}_i^1 &= p_i^1 \\
\dot{p}_i^1 &= k_2^i \tan\left(q_i^2\right) + k_3^i \sin\left(q_2^1\right)\left(p_i^1\right)^2 \\
&\quad -\left(\frac{k_i^1}{\cos\left(q_i^2\right)}\right) u_i \\
\dot{q}_i^2 &= p_i^2 \\
\dot{p}_i^2 &= u_i
\end{aligned} \tag{35}$$

where:

$$\begin{aligned}
k_i^1 &= \frac{\left(J_i^2 + m_i^2\left(l_i^2\right)^2\right)}{m_i^2 L_i^1 l_i^2} \\
k_i^2 &= \frac{g l_i^1}{L_i^1 l_i^2} \\
k_i^3 &= \frac{l_i^2}{L_i^1}
\end{aligned} \tag{36}$$

In these relations, m^j, l^j, L^j, and J^j are the masses, the lengths of the centre of mass, the lengths and the moments of inertia of the links, respectively, and their values are given in Table 1. In addition, the coefficients value of the controller are given in Table 2. By following the procedure presented in [1], Eq. (35) can be transformed into cascaded form as:

$$\begin{aligned}
\dot{x}_i^1 &= x_i^2 \\
\dot{x}_i^2 &= x_i^3\left[k_i^2 + \frac{k_i^3}{\left(1+\left(x_i^3\right)^2\right)^{\frac{1}{2}}}\left(x_i^2 - \frac{k_i^1}{\left(1+\left(x_i^3\right)^2\right)^{\frac{1}{2}}} x_i^4\right)^2 + \frac{k_i^1}{\left(1+\left(x_i^3\right)^2\right)^{\frac{3}{2}}}\left(x_i^4\right)^2\right] \\
\dot{x}_i^3 &= x_i^4 \\
\dot{x}_i^4 &= v_i
\end{aligned} \tag{37}$$

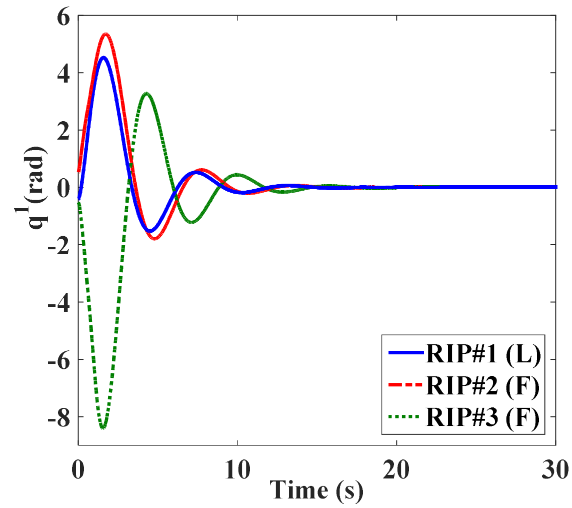
These equations are similar to Eq. (4) and can be used for applying the proposed controller. The desired values for $q_d = [q_{d1}, q_{d2}]$ are $q_d = [0,0]$. These desired values are known for leader and two followers have to follow the leader's trajectory. The output signal is transmitted to agent 2 from leader with 0.5 second delay.

Table 1. Numerical values of parameters in SI for RIPs [33]

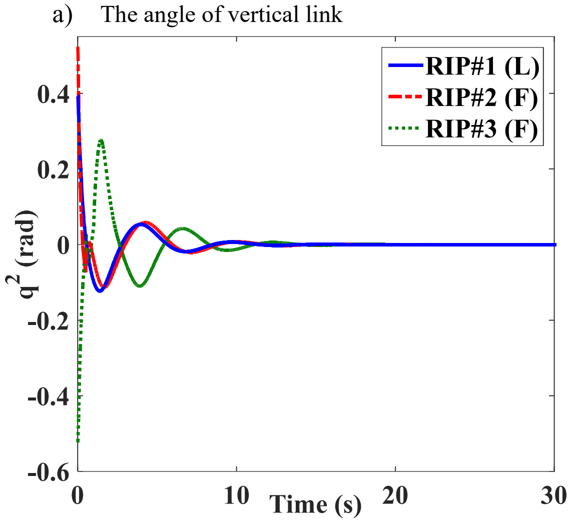
Parameters	value
Inertia of the links	$J^1 = J^2 = 1.98e^{-4}$
Mass	0.0538
Length of the links	$L^1 = L^2 = 0.215$
Length of the centre of mass to rotor	$l^1 = l^2 = 0.1075$

Table 2. Numerical values of controller coefficients for RIPs

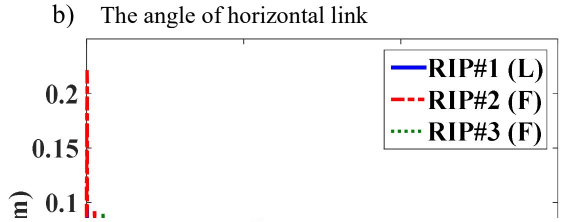
Parameters	value
$C_{11} = C_{12} = C_{13}$	3
$C_{21} = C_{22} = C_{23}$	6
$C_{31} = C_{32} = C_{33}$	3
$C_1 = C_2 = C_3$	3
$\Gamma^i = F^i$	I_6



a) The angle of vertical link



b) The angle of horizontal link



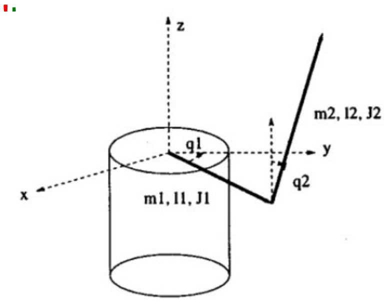


Fig. 3. *Rotary inverted pendulum [1]*

Fig. 4. The performance of the proposed controller in tracking the consensus value for RIP example

5. Quadrotor

The dynamic equations of the i th quadrotor (Fig. 5) can be represented by the set of equations [32]:

$$\begin{aligned}
\ddot{x}_i &= \frac{1}{m_i}\left(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i\right)u_i^1 - \frac{c_i^1 \dot{x}_i}{m_i} \\
\ddot{y}_i &= \frac{1}{m_i}\left(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i\right)\mathrm{u}_i^1 - \frac{c_i^2 \dot{y}_i}{m_i} \\
\ddot{z}_i &= \frac{1}{m_i}\left(\cos\phi_i \cos\theta_i\right)u_i^1 - g - \frac{c_i^3 \dot{z}_i}{m_i} \\
\ddot{\phi}_i &= \dot{\theta}_i \dot{\psi}_i \frac{I_{yi} - I_{zi}}{I_{xi}} + \frac{J_{ri}}{I_{xi}}\dot{\theta}_i \Omega_{ri} + \frac{l_i}{I_{xi}}u_i^2 - \frac{c_i^4 l_i}{I_{xi}}\dot{\phi}_i \\
\ddot{\theta}_i &= \dot{\psi}_i \dot{\phi}_i \frac{I_{zi} - I_{xi}}{I_{yi}} - \frac{J_{ri}}{I_{yi}}\dot{\phi}_i \Omega_{ri} + \frac{l_i}{I_{yi}}u_i^3 - \frac{c_i^5 l_i}{I_{yi}}\dot{\theta}_i \\
\ddot{\psi}_i &= \dot{\phi}_i \dot{\theta}_i \frac{I_{xi} - I_{yi}}{I_{zi}} + \frac{1}{I_{zi}}u_i^4 - \frac{c_i^6}{I_{zi}}\dot{\psi}_i
\end{aligned} \tag{38}$$

Here x, y, and z are three states corresponding to translational motion along three coordinate axes. φ, θ, and ψ are roll, pitch, and yaw angles, respectively.

The simulation results of applying proposed controller on the quadrotor's under-actuated subsystem are provided in Fig. 6. The initial and desired values for states of under-actuated subsystem for 3 agents are, $[x_1; y_1; \varphi_1; \theta_1] = [0;0;0;0]$, $[x_2; y_2; \varphi_2; \theta_2] = [-0.5;-0.5;0.01;0.01]$, $[x_3; y_3; \varphi_3; \theta_3] = [-0.3;-0.3;0.02;0.02]$, and $[x_d; y_d; \varphi_d; \theta_d] = [1;1;0;0]$, respectively. The communication topology is subjected to 0.5-second time delay, as well. As shown in Fig. 6, the follower agents drive to the unknown consensus value by the proposed method, very quickly.

In order to investigate the robustness of the controlled system against uncertainties in system parameters, the values of moments of inertia in the controller are selected as $I_x = I_y = 2$ and $I_z = 1.5$ instead of their real values in

Table 3. The performance of the controller is demonstrated in Fig. 7. As can be seen, the proposed controller is robust against uncertainties in system parameters and the agents come to an agreement, rapidly.

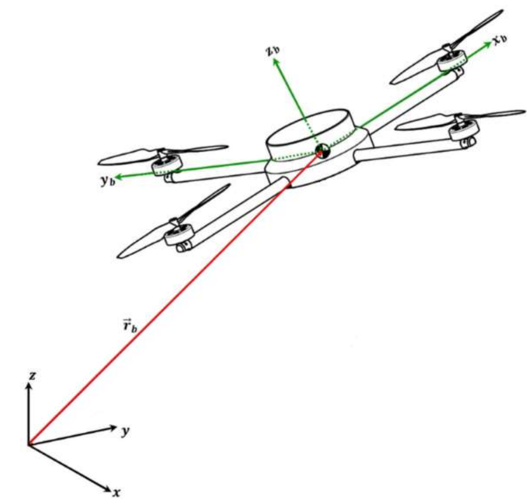


Fig. 5. Schematic of a quadrotor

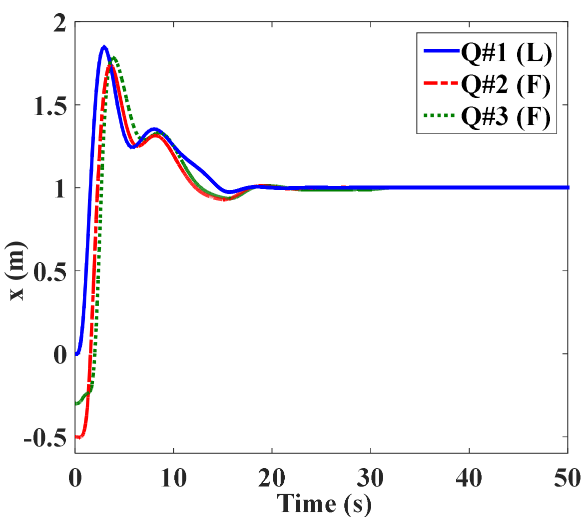
Table 3. Numerical values of physical and geometric properties in SI for quadrotors [32]

Parameters	value
Moment of Inertia	$I_x = I_Y = 1.25$, $I_Z = 2.2$
Mass	2
Drag Coefficient	$C^1 = C^2 = C^3 = 0.012$ $C^4 = C^5 = C^6 = 0.012$
Rotor Distance to Center of Mass	$l = 0.2$

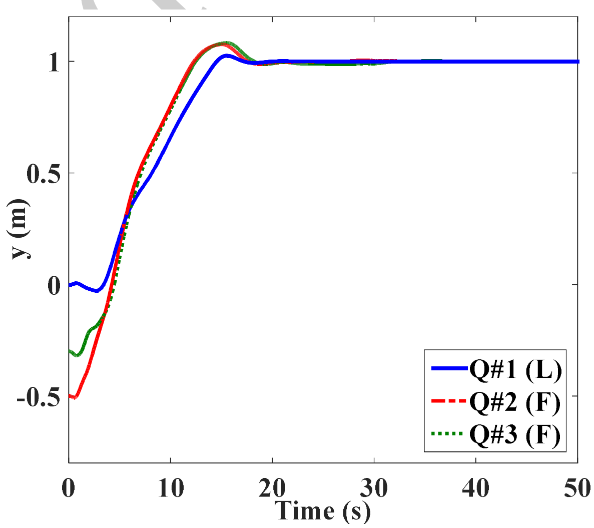
Table 4. Numerical values for controller coefficients for quadrotors

Parameters	value
β_1	$12l_2$
β_2	$10l_1$
λ	3
α	0.3
η	1
b	0.01

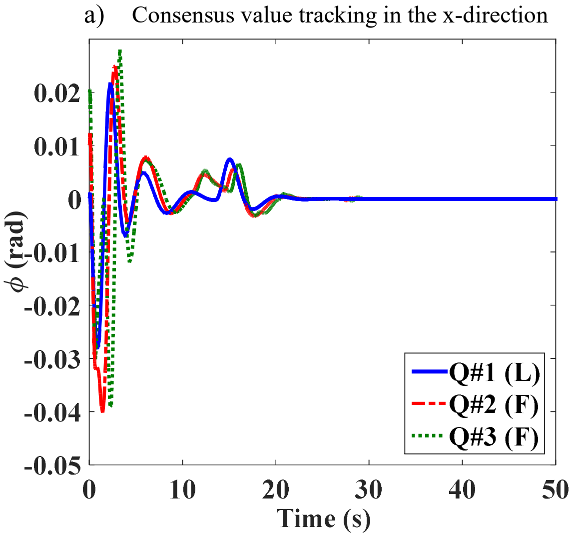
In order to make comparison between the results of the proposed controller performance and the conventional AFSMC, Fig. 8 has been drawn. Qualitatively, the proposed controller has improved the performance of the system in the transient part of the response,



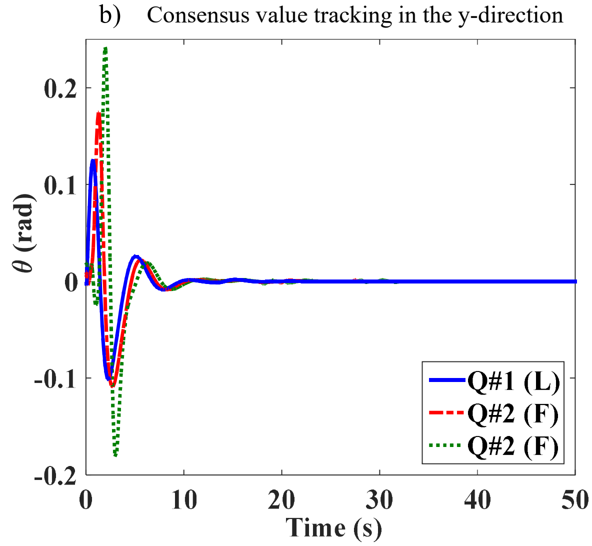
a) Consensus value tracking in the x-direction



b) Consensus value tracking in the y-direction



c) Consensus value tracking about x-axis



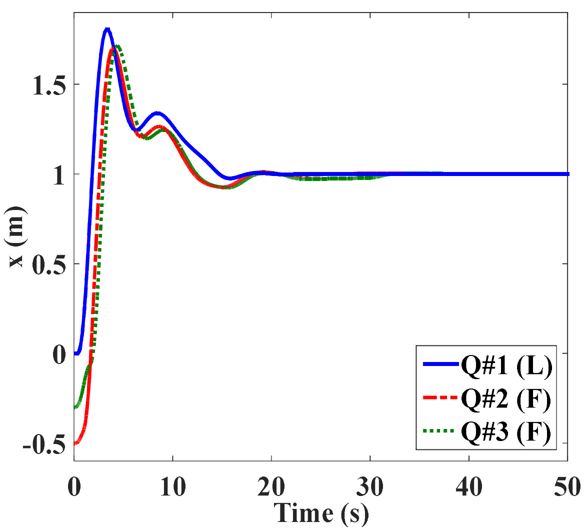
d) Consensus value tracking about y-axis

Fig. 6. The performance of the proposed controller in tracking the consensus values for quadrotor example when the system dynamics is exact

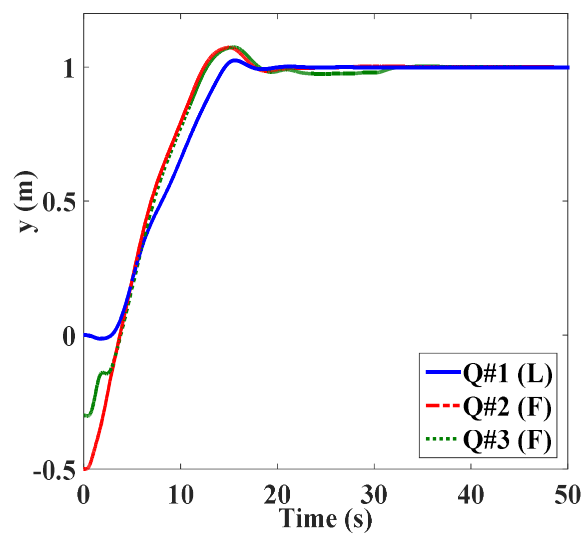
considerably. In the conventional AFSMC, the agents move to their desired values after a drop in the response while in the proposed AFSMC the agents drive to their consensus values, immediately. Quantitatively speaking, the reduced percentage of rise-time for agents 1, 2, and 3 in the proposed controller is about 30, 12.6, and 16.1, respectively in comparison with the conventional AFSMC. Furthermore, the reduced percentage of maximum overshoot is 0.04, 1.4, and 1.7 for agents 1, 2, and 3, respectively. The maximum rise-time is reduced to 1.6144 from 2.233. Also, the maximum overshoot is reduced to 1.7163 from 1.7455. These explanations show the superiority of the proposed controller in transient part of the response, especially rise-time, in comparison with the conventional AFSMC.

6. Conclusions

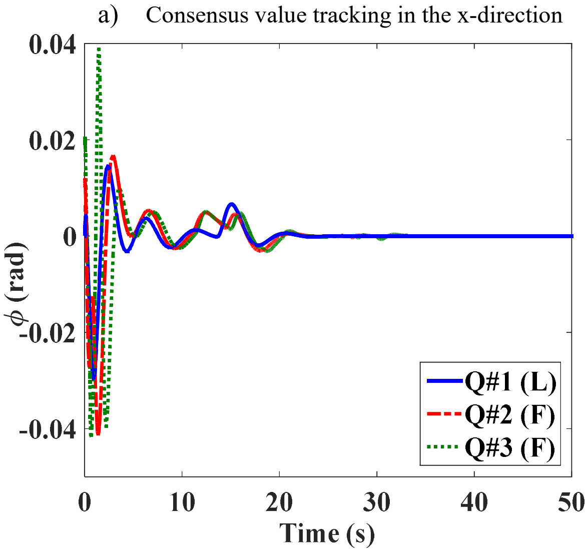
An improved AFSMC for synchronization of networked UASs under unknown communication delay is presented in this paper. The equivalent part of input control has been estimated by a TSK fuzzy system and the robust part of the controller has been modified in order to synchronize the states of the agents in the network. The results of the simulation show that by using the enhanced controller the follower agents, in a leader/follower paradigm of information network, converge to leader's state in the presence of unknown communication delay, very quickly. In addition, the robustness of the enhanced controller against uncertainties has been shown through simulations. The most important feature of the proposed strategy is the improvement of the properties corresponding to the transient part of the response in comparison with the conventional AFSMC. As shown, significant improvement in reduction of rise-time of the response



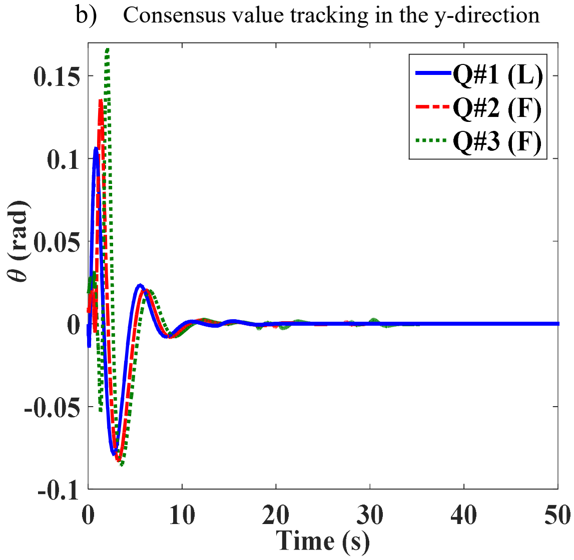
a) Consensus value tracking in the x-direction



b) Consensus value tracking in the y-direction

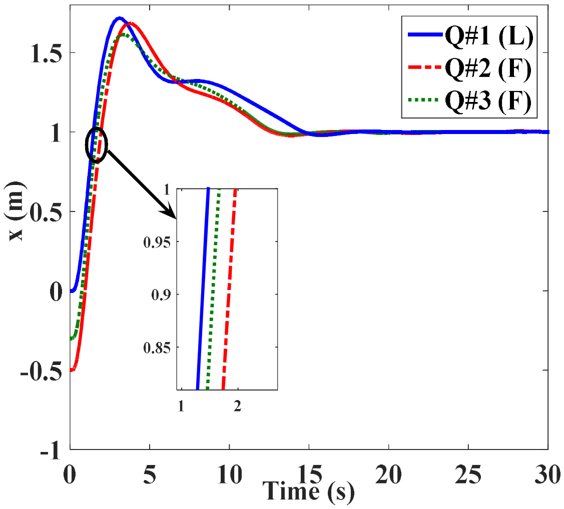


c) Consensus value tracking about x-direction

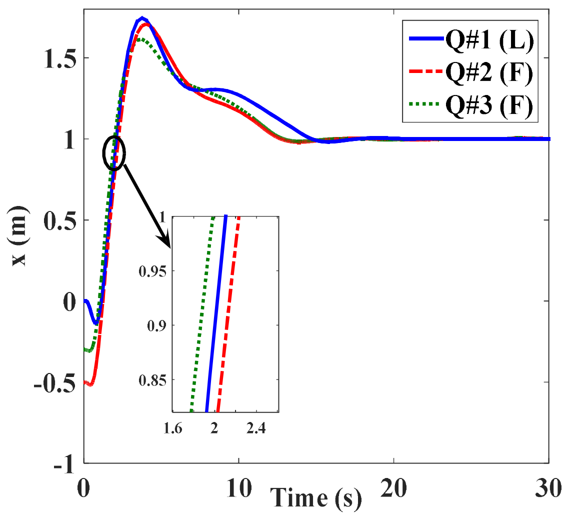


d) Consensus value tracking about y-axis

Fig. 7. The performance of the proposed controller in tracking the consensus values for quadrotor example when the uncertainties in moments of inertia exist.



a) Proposed AFSMC



b) Conventional AFSMC

Fig. 8. Comparison of the proposed synchronization AFSMC with the conventional AFSMC.

is achieved by the proposed AFSMC.

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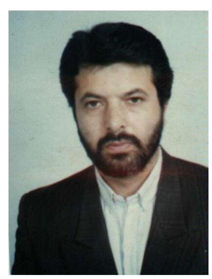

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